DOCUMENT RESUME

ED 130 828 SE 020 622

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TITLE The Laboratory Approach to Math Instruction. Module CMM:006:04.
INSTITUTION Idaho State Univ., Pocatello.
PUB DATE Sep 72
NOTE 71p.; For related Modules, see SE 020 618-624; Not available in hard copy due to marginal legibility of original document

EDRS PRICE MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS Activity Learning; Curriculum; *Elementary School Mathematics; Elementary School Teachers; Higher Education; Instruction; *Instructional Materials; *Laboratory Procedures; Mathematics Education; Methods Courses; *Performance Based Teacher Education; *Preservice Education; Teacher Education; Teaching Methods

ABSTRACT

This fourth module in a sequence of instructional modules for preservice elementary teachers deals with the laboratory approach to mathematics. The module focuses on laboratory activities aimed at readiness, concept development, concept synthesis, recall, application, planning, evaluation, and remediation. Objectives for the module, prerequisites, pre- and post-assessment standards, and remediation activities are listed along with 9 instructional activities for the preservice teachers to complete. Eleven reading handouts covering laboratory organization, instructional approaches, and instructional materials are included. (DT)

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CMM:006:04(ISU)

The Laboratory Approach to Math Instruction

Objectives:

At the completion of module study, the Intern will be able to:

1. Identify the characteristics unique to instruction in math which utilized The Laboratory Approach. (KNOWLEDGE)

2. Distinguish learner activities which illustrate the LAB approach from those which do not. (KNOWLEDGE)

3. Identify and Demonstrate examples of Learner Laboratory activities in math which are needed to support the following: (KNOWLEDGE AND PERFORMANCE)

   a. Readiness
   b. Concept Development
   c. Concept Synthesis
   d. Recall
   e. Application of knowledge
   f. Planning, evaluation, and remediation.

4. Visit at least 5 classrooms other than the ones you have worked in and observe math instruction including: (EXPRESSIVE)

   a. Use of space
   b. Use of math materials and facilities
   c. Teaching-learning modes employed
   d. An informal discussion about math with at least 3 teachers and 3 learners in each of those teachers' rooms.

5. State a rationale for The Lab Approach including: (KNOWLEDGE)

   a. A description of a Theoretical Basis for The Laboratory Approach
   b. A description of a set of standards for space and facilities which are needed to implement The LAB Approach with a group of learners.
   c. A description of the teacher roles and the learner roles to be explored in a LAB Approach.

6. Identify the "elements" (according to KIDD) of a math LAB Guidesheet (or Activity Card) and design and demonstrate a series of guidesheets which includes ALL the "elements". (KNOWLEDGE AND PERFORMANCE)

7. Design and demonstrate a "Laboratorizing" of some math learning, including: (PERFORMANCE AND CONSEQUENCE)

   a. A description (written or sketched) of the space and facilities utilization you devised.
   b. A description of the "implementation plan" you employed in the "Laboratorizing" process.
   c. An Observation Form or Procedure which you designed to monitor Learner Involvement in the LAB Approach.
   d. Results of data which you collected from the Observation Form or Procedure which describes "Learner Actions" while in the LAB Activities.

SE 020 622

The Laboratory Approach to Math Instruction

Pre-Requisite:

Completion of modules: CMM:006:01 (ISU) through CMM:006:03(ISU)

Pre-Assessment:

1. Intern will complete the Pre-Assessment covering the knowledge portions of Objectives. Minimal Performance Levels are indicated in the Pre-Assessment.

2. Intern will demonstrate Performance portions of Objectives 3, 6, and 7.

3. Intern will complete Objective 4.

Instructional Activities:

1. Read and utilize the following:
   a. Kidd: The Laboratory Approach to Mathematics, (NOTE: Since this entire volume attempts to modelize the LAB Approach, the Intern should selectively read and study materials throughout the volume which relates to the Objectives of this module.)
   b. Davidson: Using the Cuisenaire RODS--A Photo-Text Guide for Teachers, read p. 5 "How to Use This Material" and scan and explore the remainder of the Guide briefly at this time.
   c. Dumas: Math Activities for Child Involvement, read "Math Activities and You", pp. 3-22 and briefly scan the entire volume at this time.
   d. HANDOUTS:
      1. "The Laboratory Approach to Elementary Mathematics"
      2. Beuthel and Meyer: "A Regular Classroom Plus a Mathematics Laboratory"
      3. "Criteria For Hands-On-Learner-Orientation in Mathematics" (NOTE: especially read and attempt to impletment the brief section on page 2 "Your Task and Mine".)
      4. Grossman: "Problem Solving Activities Observed in British Primary Schools".
      5. "Why Manipulative Materials?"
      6. Kramer: "The Role of Discovery: Math"
      7. "Asking Challenging Questions"
      8. Lanning: "Using Pupil Teams in Teaching" (NOTE: This handout includes basis suggestions for implementing a Lab Approach through use of pupil teams.)
      9. Davis: "The Needs of the Curriculum and the Needs in Teacher Education"
      10. Handgrove and Sueltz: "Suggested Instructional Materials" (NOTE: This handout will be helpful in your attempts to explore the LAB Approach.)
      11. Davidson: "An Annotated Bibliography of Suggested Manipulative Devices" (NOTE: This handout is probably the best single collection to date of the wide variety of manipulatives which support labizing efforts in math. Some of these may be available within your target schools--try to locate them--some are simple and may be made inexpensively; a few are available on short loan from the ISU/IMC, and some you may wish to negotiate for purchase.)

d. HANDOUTS cont'd) (NOTE: There are many other sources for LAB Approach ideas and attempts. Be alert to the sources in your target schools and as you identify useful items, you are urged to share this information with the instructor when you send in the module evaluation sheets. As the instructor either develops or becomes aware of additional items, these will be shared with you to use as you see fit.)

2. Utilize ideas in the The State Math Guide, design a series of math lab guidesheets or activity cards (including a teacher's portion to the series of guidesheets or activity cards. The Addenda section of the math guide has several specific ideas and materials for developing guidesheets or activity cards.

3. Explore the materials and resources available in the ISU/IMC and check out (for short loan) and utilize some materials or devices in your labizing efforts.

4. Devise a procedure to encourage learners to design their own guidesheets or activity cards. Their results could be "deposited" into your labizing efforts. This is an especially valuable activity as learners (and you) become more "tuned" to the LAB Approach and when acceptance by cooperating teachers and learners become more apparent. The team approach to learners developing games and guidesheet activities can become one of the most rewarding experiences in math for all concerned.

5. Design an instrument or procedure (related to Objective 3) which will assist you in reporting your Demonstration Performances. Such an instrument or procedure might be in the nature of a set of activity cards or guidesheets which you designed and implemented with learners which supported the:
   a. readiness
   b. concept development
   c. concept synthesis
   d. recall
   e. application of knowledge and
   f. planning, evaluation, and remediation

   (NOTE: The Kidd volume, The Laboratory Approach to Mathematics, pp.4-23, will be especially helpful in completing this Objective.)

6. Visit classrooms to observe math instruction and learning (SEE: Objective 4). As you involve yourself with such visits, you will find it useful to note:
   a. how the teachers and learners use the classroom space during math
   b. how they make use of various materials and facilities in learning math, and
   c. the teaching-learning Modes employed, (SEE: Module CMM:006:03), and involve yourself in talking informally with teachers and learners about math.

   NOTE: Objective 4 is "Expressive", that is, there is nothing that you are expected to learn or demonstrate as a result of your experiences with these visits. The experiences will have different meanings and impacts upon different Interns, but it will allow you to check what you see in these classrooms with what you are being asked to demonstrate within this module. Some teachers may put so much emphasis on the "getting through of material" that they are unable to explore "how children learn and experience the quantitative world around them".

7. Your efforts in this module will be continued in the next module CMM:006:05(ISU) "Math Lab Kits and Activities". In fact, exploration of that module should almost accompany the study of this module since it is designed mainly as a "support system" for your efforts to LABIZE your invovlement with the teaching of math. If you have not received that module, you may request it and work these two somewhat simultaneously. The ACTIVITY CARD especially is introduced during the next module.

8. Develop a plan for "LABORATORIZING" some math learning with a group of learners (small group or entire class group). Try your plan on with your Team Leader, other Interns, Cooperating Teacher, or even the Principal. You are especially encouraged to develop your plan "cooperatively" with other Interns.

   Implement your plan for "Laboratizing". Include designing an observation form or procedure for monitoring learner involvement the LAB approach; and collect, summarize, and report data which describes the "Learner actions" while participating in the LAB activities. (NOTE: This responds to Objective 7 primarily, but will assist you in satisfying other objectives as well. You will want to do your "homework" in terms of your plan. You will also want to realize that the LAB Approach, though not new, will provide a source of concern and anxiety for some teachers, administrators, and even you. It requires exploring new teacher roles (primarily "Inquiry Mode") and new learner roles. If you have done your "homework in terms of the Rationale and Theoretical Basis for the LAB Approach", you will weather the storms. If you don't build this base first, you will likely experience resistance to the concepts involved. If you are not grounded in the "WHY" of what you are attempting as well as the "HOW", you have little basis for suggesting or attempting to change math instruction and learning.

9. Design and carry out your own Instructional Activities related to the objectives of the module. There are no rights or wrongs in terms of attempting a LAB Approach if you operate within some of the rationale and theoretical framework of "Labizing". Your efforts should move away from drill in facts and computation, expository lessons, and move toward a "hands-on point-of-view".

Post-Assessment:

1. Intern complete Post-Assessment covering the knowledge portions of objectives. Minimal Performance Levels are indicated in the Post-Assessment.

2. Submit evidence of completion of the Performance, Consequence, and Expressive Objectives.

   a. Objective 3: Submit the Instrument or Procedure which you developed in such an Instructional Activity as #5. This might be a set of Activity Cards or Guidesheets which illustrate the factors listed in Objective #3.

   b. Objective 4: Submit a Log statement listing what you did. Since this was an Expressive Objective, you need not evaluate it per se.

   c. Objective 6: Submit a series of Guidesheets which includes the "elements" of a good guidesheet as described in Kidd. (NOTE: This could also be used as a portion of Objective #3.)

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d. Objective 7: Submit your plan, your observation form or procedure, and the results of your use of the form or procedure in terms of learner actions due to the LAB activity.

NOTE: The above items should be checked with the Team Leader and then sent to the Instructor.

Remediation:

This should be determined in consort with the Team Leader and/or Instructor. If an Instructor seminar is needed, the Team Leader should notify the Teacher Corps Office.

MODULE RECORD FORM

Intern_____

Module No._____Title:_____

Date Started:_____

Date Completed:_____

Record of Team Leader Conferences: (Team Leader please initial)

Dates	TL Initials

Record of Instructor Seminars: (Instructor please initial)

Dates	Inst. Initials

Written materials completed. (Please attach Pre- and Post-Assessments and written records on Instructional Activities.)

Send this form and attached materials to the Teacher Corps Office.

For TC Office use ONLY:

Included in _____course credit.

Date Module Satisfied_____Approval_____

REMARKS:

MODULE FIELD-TEST EVALUATION FORM

INTERN

MODULE #_____ TITLE_____

COURSE #_____ MODULE BEING STUDIED BY_____
(INTERN)

---

1. INDICATE WHICH CATEGORY YOU BELIEVE BEST DESCRIBES YOUR EVALUATION OF THIS MODULE: (CHECK ONE)

____A. KEEP-AS-IS
____B. DELETE ENTIRELY
____C, MODIFY BY DELETING PORTIONS (UNDER #2 IDENTIFY WHICH PORTIONS SHOULD BE DELETED.)
____D. MODIFY BY ADDING PORTIONS (UNDER #2 IDENTIFY WHICH PORTIONS SHOULD BE ADDED.)
____E. MODIFY BY RE-WRITING OR RE-DESIGNING PORTIONS (UNDER #3 IDENTIFY THE KINDS OF RE-WRITE OR RE-DESIGNING YOU VIEW NEEDED--BE SPECIFIC.)
____F. OTHER (SPECIFY UNDER #2).

2. SUGGESTIONS FOR MODIFICATIONS OF MODULES: (FROM #1--C, D, E, OR F ABOVE).

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3. HOW DOES THIS MODULE SATISFY THE FOUR CRITERIA FOR A MODULE LISTED BELOW? (YOUR COMMENTS ARE WELCOMED).

	NOT AT ALL	LITTLE	SOME	VERY WELL
A. "FIELD-ORIENTED"--(UTILIZES KNOWLEDGES AND SKILLS IN APPLICATIONS WITH LEARNERS IN THEIR USUAL LEARNING ENVIRONMENTS IN THE SCHOOL AND COMMUNITY):	_____	_____	____	_____

COMMENTS:

B. "COMPETENCY-BASED"--(REQUIRE DEMONSTRATION BY INTERN OF CRITERION-REFERENCED COMPETENCIES IN KNOWLEDGES, PERFORMANCES, SKILLS, ATTITUDES, ETC.)	_____	_____	____	_____

COMMENTS:

MODULE FIELD-TEST EVALUATION FORM

INTERN

	NOT AT ALL	LITTLE	SOME	VERY WELL
C. "PERSONALIZING"--(ALLOWS INTERN TO CHOOSE FROM AMONG ALTERNATIVE INSTRUCTIONAL ACTIVITIES INCLUDING DESIGNING HIS/HER OWN, RELATED TO THE OBJECTIVES OR COMPETENCIES OF THE MODULE):	_____	_____	___	_____
COMMENTS				
D. "EMPHASIZE SATISFACTION OF PRODUCT OR CONSEQUENCE OBJECTIVES"--(EMPHASIS UPON LEARNER PERFORMANCE OUTCOMES--WHAT CHANGES TAKE PLACE IN CHILDREN AS A CONSEQUENCE OF THE INTERN'S INSTRUCTIONAL EFFORTS):	_____	_____	___	_____
COMMENTS				

TC OFFICE

_____

MAIL OR SEND TO TC OFFICE I.S.U., POCATELLO

DATE EVALUATED: _____

EVALUATORS NAME _____

CHECK ONE:

INTERN _____
TEAM LEADER _____
COOPERATING TEACHER _____
INSERVICE TEACHER _____
OTHER _____ (IDENTIFY)

THE LABORATORY APPROACH TO ELEMENTARY MATHEMATICS *

Elementary teachers are finding the laboratory approach an effective means of individualizing the teaching of mathematics. The approach is useful in motivating children to learn, in providing them with more than one way of learning the same material, and in fitting methods to the child's learning style.

The laboratory approach exposes children to a wide range of manipulative, concrete materials and practical activities from which they can abstract mathematical ideas. Each child has the opportunity to learn individually--at his own rate--and to work with a buddy (or a small group) as he explores and discovers mathematical relationships. Both teachers and students experience delight in an approach that emphasizes individual learning--in settings that foster discovery and lead children to experience math that not only makes sense to them but is fun to learn.

An elementary school mathematics laboratory doesn't have to be an all or none affair. It may start with a few well-chosen materials arranged on a table and shelves in a corner of the classroom.

Of course, it would be highly desirable to have a special laboratory room with its own mathematics equipment--and ample storage space where materials can be categorized, labeled, and made accessible to teachers and students. Such a room should have lots of open space, with furniture (mainly tables and chairs) that can be easily moved about. However limited or extensive the lab may be, it should be operated with the full cooperation of the school's library resource center, thus ensuring that films, filmstrips, tapes, and supplementary books and reference materials will be readily available.

The teacher who sets up a mathematics laboratory has at least three general types of materials in mind. The first type is largely motivational and encourages children to learn mathematics. The second type provides horizontal enrichment of the basic program for each child at his level. The third type offers vertical enrichment of the kind that enables the child to work on extensions of topics included in the program and on selected topics outside the program.

Games, mathematical puzzles, and tricks are examples of the motivational type of material. Some games require children to build number sentences and equations (Tuf, Equations) while others provide practice in operations with whole numbers and fractions (Number Bingo, Orbiting the Earth). Examples of mathematical puzzles include the Tower of Hanoi (or pyramid puzzle); the Chinese shuttle (or strip puzzle); and Tangram puzzles, in which two or more pieces are fitted together to form a square, triangle, or hexagon. Puzzle games (Twenty-One, Nim) fascinate young and old alike.

Shortcuts for adding and multiplying, schemes for finding a person's age and the month in which he was born or for telling the number a person is thinking of, and the arrangement of numbers in magic squares may look like tricks to the observer. However, with the use of cue cards which take children through a sequence of activities, a student may discover the rules and principles on which puzzles and tricks of this sort are based. Logical thought, practice on number facts, and discovery of mathematical principles and relationships are highly desirable concomitant values to the major purpose of stimulating and maintaining children's interest in learning mathematics.

* From: Today's Education, February, 1971.

The Laboratory Approach to Elementary Mathematics

Horizontal enrichment materials supplement the program by providing concrete experiences when a new topic is introduced. Their function is to extend the development of a concept, or to offer a different way of approaching a topic.

Two examples of horizontal enrichments for young children are attribute blocks or similar materials varying in size, color, and shape which are used in working with sets, and the Piaget-type activities which guide children to form concepts of conservation of number and counting, of order and reversibility. These are essential prerequisites to learning the facts of the operations.

For slightly older children in the elementary school, materials already in use by some teachers could qualify: rods, beads, and blocks based on a standard unit (Cuisenaire rods, Dienes blocks), counting frames, hundred boards, place value materials, flannel boards, and cutouts--for work with whole numbers, fractions, geometry, and problem solving--as well as a magnetized section of the chalkboard with a supply of magnetic discs and other shapes to use on it.

The teacher should select horizontal enrichment materials to help individual pupils overcome specific difficulties and to reteach topics that are preventing them from making progress in mathematics. This implies starting each pupil at his level of understanding. It also implies diagnosis to uncover gaps in his learning, and remediation through experiences with materials that will help close those gaps.

Assigning a pupil a lot of drill work of the same type that has already given him difficulty is not the answer to his problem. What he really needs is an opportunity to learn by means of a new approach with different materials. And most of all he needs sustained encouragement that will stimulate him to keep working at a task that he feels he can do and wants to do.

These children must also have experience with materials of the motivational type and much experience with a variety of horizontal enrichment materials described earlier. How dull it would be for a child to spend all of his laboratory time in phases of mathematics with which he already feels defeated!

Vertical enrichment consists of providing openended, ungraded materials that encourage the child to go as far as his ability and interest enable him to go with a specific topic. These materials are often developed around areas of mathematics that teachers feel receive skimpy treatment in the regular program and that therefore make it difficult for able students to explore their special interests.

For example, to enhance the study of geometry, the laboratory contains such materials as geoboards (nailboards), rubber bands for enclosing geometric shapes, cards or work sheets to guide the pupil individualized learning, and dot paper simulating the geoboards so that he has a permanent record of his discoveries.

Mirror cards and unbreakable mirrors can stimulate a study of mirror reflections and learning about lines of symmetry. Units developed from current space exploration and sports events, from weather conditions, and other environmental happenings in the news--or units combining either mathematics and science or mathematics and social studies--may bring new insights in mathematics and its relationship to other fields.

Other types of material for vertical enrichment include the Shoe Box Kits developed under the direction of Robert B. Davis of Syracuse University, director of the Madison Project which was partially sponsored by the U.S. Office of Education, and learning packages and sets of activity cards which are now available on a number of mathematical topics.

Of course, some materials, such as activity cards, serve more than one purpose. They may either be purchased or prepared by teacher and pupils. For example, the following activities require a sheet of one-inch squared paper, pencil, scissors, and ruler.

Card 1: Place your foot on the paper and trace around it. Cut out the pattern. Determine the number of whole, one-inch squares in your pattern. Put all the pieces of squares together and find out about how many whole squares they will make. Write up what you did and how you found your answer.

Card 2: Use your ruler to measure your foot pattern the long way to the nearest inch. This is your foot length. Use your foot length to measure five things in the room. Then use your ruler to measure the same five things, and write up your findings.

If funds for the laboratory are limited, homemade games for providing practice in computation are often highly satisfactory substitutes for commercial products. For example, a simple miniature tenpins game can be made from 10 spring-type clothespins, a small sponge ball, and a runner marked to show starting line at one end and circles at the other arranged to indicate placement of the pins.

A game resembling a commercially produced one can be made from an egg carton. First, write the numerals from 1 through 12 in the partitions of the carton. Use colored, see-through discs to cover the numerals.

A child throws two dice and then covers either a single numeral in the egg carton that represents the sum of the throw or any two numbers that add up to the sum. He continues until he can no longer cover any of the remaining numerals or until all numerals are covered. (The latter rarely happens.)

The player finds his score by adding the numbers which he has been unable to cover. Since a low score is desirable in this game, each child tries to figure out the strategy of covering numbers that will enable him to get the lowest possible score.

Children enjoy collecting materials for the laboratory, and they are often able to assist in categorizing and classifying them for later use. One day they might concentrate on bringing in materials to use as counters: buttons, acorn caps, bottle tops, small stones, and large seeds. Another day they might bring containers made in various geometric shapes--cube, cylinder, and the like. Children also enjoy collecting pictures that illustrate number facts; maps to use in determining distances; and cloth scraps and wallpaper patterns with repeating designs for use with mirror work.

Pupils receive further experience by classifying materials. For example, they can sort buttons according to the number of holes (0,2,4), according to size (small, medium, large), or according to color. Thus, a button collection serves as a satisfactory substitute for attribute blocks.

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Once they actively participate in setting up a laboratory, pupils begin to view their surroundings with a mathematical eye. The paper in daddy's adding machine is a natural material for making number lines; mother's kitchen equipment may be useful for the measurement display; throwaways around the house (boxes, cartons, blocks, old clocks) are also useful.

A listening center serves many purposes in the mathematics laboratory. A teacher may prepare practice tapes following diagnosis of the facts a pupil needs to practice. On the basis of written and oral interview testing, each child would have his own grid table of addition or multiplication. (The facts he already knows are blocked out immediately, and his practice tape is made of the remaining facts.)

The same tables can be used for subtraction and division. To make the tape, read each fact onto the tape--allowing sufficient time after each item for the child to respond orally or in writing. This becomes the child's own tape; he may listen to it as often as necessary. As a pupil works on his practice tape and sees the task becoming smaller and less formidable, he is encouraged to keep working for accurate responses until all facts are blocked out.

Tapes may also help children with reading disabilities which interfere with their success in mathematics. Instructions for work sheets and activity cards may be put on tape, until the child develops better reading skills.

Classroom teachers can use a special laboratory room in different ways. Part of the class may have a laboratory period, while the regular teacher works with the rest of the children.

Good students who finish assignments and have time to spare may find greater challenge from laboratory offerings than most classrooms are able to provide. Under no circumstances, however, should the laboratory be reserved only for children who finish assigned work early. Weaker students are stimulated to greater effort as they become successful with individualized activities in the laboratory and begin to grasp some idea of the scope of mathematics.

How to evaluate individual laboratory procedures and results is a persistent problem that challenges the regular classroom teacher and the laboratory teacher alike.

One teacher uses a weekly period when half the class is out of the room for checking the children's progress and for planning future laboratory activities. Some teachers use a file folder for each child with record sheets for checking off work accomplished (games played, contests, work sheets, and activity cards completed). These are checked by the teacher and pupil, as they plan together for the child's needs.

A diary-type record in which the child describes his project, and what he learned from it, is a good self-evaluation device. The possibilities for integrating mathematics with the language arts program are apparent in this approach, for the child must organize his thoughts, plan his work, and express his ideas in sentence form.

Certainly, some aspects of a laboratory approach are feasible for any teacher. As he becomes more comfortable with the approach he has chosen, he may wish to include more mathematics topics, devote more time to them, or even to expland the program to include subjects other than mathematics.

A REGULAR CLASSROOM PLUS A MATHEMATICS LABORATORY*

Donald G. Beuthel

and

Phyllis Meyer

For years we taught arithmetic using "just a textbook: with the few variations of games, visual devices, and limited activities that could be used to individualize instruction in the confines of a single classroom. It was with a skeptical, yet receptive, attitude that we first read "A Mathematics Laboratory--From Dream to Reality" by Patricia S. Davidson and Arlene W. Fair in the February 1970 issue of the Arithmetic Teacher. We wanted to try something like what was described there, but the major problems that kept popping up in our minds were very real ones: a curriculum system and guidelines that strongly suggested a textbook in every child's hands; a local school community that had always ranked high in test-score achievement and thus demanded a somewhat rigid, disciplined program including weekly homework; no empty classrooms and no other space available to begin such a laboratory; no money for the new materials and devices that would be needed. Finally, our own schedules of teaching left no time for either of us to devote to such a project during the school day.

In spite of these obstacles, we have been able to put into operation the beginnings of a program that is not an "either-or" one, but an "and"--a mathematics laboratory in addition to our normal classroom activities. Each of these programs has its own benefits. For some children, one approach is better than the other. The textbook and the mathematics laboratory approaches complement each other. Together they provide a broader and more far-reaching program, thereby permitting a greater range of experience for our students.

We read with particular interest the two articles in the May, 1971 issue of the Arithmetic Teacher, "What's Your Position On....The Role of Experience in the Learning of Mathematics?" by Edith E. Biggs and by Maurice L. Hartung. We can see in our program the three aspects of mathematical experiences--physical, mental, and emotional--mentioned by Hartung. We also think that our program includes such types of discovery learning as impromptu discovery, free exploratory discovery, guided discovery, and programmed learning mentioned by Biggs. We happened on most of these things through trial and error in the process of establishing our mathematics laboratory, and we can testify to the value of all of these ideas. Within a program stressing individualized instruction, we have seen that each child requires a variety of approaches to best meet his individual needs.

Our particular program began nearly two and one-half years ago when we attended a demonstration of devices that could be used in a mathematics laboratory. A few months later, we had an opportunity to visit a mathematics laboratory in a local private school that had a smaller pupil-teacher ratio, more flexible scheduling and

*Arithmetic Teacher: November, 1972; pp. 527-530.

fewer ties to tradition than prevailed in our own teaching situation. Suddenly the thought came to us: Couldn't we adapt this idea to our school? We discussed our ideas with the principal and were given the approval to proceed.

We immediately encountered three problems: no space, no staff, and no money for supplies and equipment.

The space problem was solved when a 9-foot by 13-foot storeroom was found on the second floor of our school building. This provided ample shelf space for equipment and enough floor space for three tables. After considerable rearranging and sorting, unnecessary things were transferred to other storerooms. Then tables and chairs were moved in, a magnetic chalkboard and pencil sharpener were added, and our project had begun. Just short of two weeks later, our mathematics laboratory was open for parents to visit on back-to-school night.

Obviously, such a small room did not lend itself to large group use. Therefore, we structured our program around individual or small group use.

A mathematics laboratory, as we see it, can fill a variety of needs. It is a place where:

1. A child may go to research a problem and investigate independently.
2. A child can find challenging instructional activities, including puzzles and games.
3. Children can help one another (children continually reinforce their own learning when they explain concepts to others).
4. Equipment for mathematical experiments is stored.
5. Various media can be used to help teach, reteach, and reinforce mathematical concepts.

To staff the mathematics laboratory, we recruited volunteers through the Parent-Teachers Association. We put a notice in the newsletter asking for volunteers to serve for one to two hours per week. The response was more than we had expected. In a short time, we had twenty-six math-lab aides, as our parent volunteers were called. We planned an orientation meeting for math-lab aides. We prepared a twelve minute slide-tape presentation entitled "We Need Your Help" to convey our ideas of a mathematics laboratory and to describe exactly what a math-lab aides responsibilities would be. We explained that a volunteer worker is an aide, not a teacher. His function is to provide guidance and supervision. The qualifications needed are: (1) a vast amount of patience, (2) a genuine interest in helping children, (3) a minimum knowledge of simple mathematics, and (4) an interest in an orderly learning procedure. An aide would be expected to arrive promptly; learn to operate the audiovisual equipment he would be using; and at the end of his assigned time, assist the children in seeing that all materials are returned to their respective places (everything was color coded so that it could easily be put back where it belonged). By the end of the school year, we had a regular staff of nineteen aides plus ten substitutes. Each aide was responsible for getting his own

substitute if he was to be absent at any time. The substitutes were trained in the same manner as the aides; so we always had a trained volunteer in our laboratory. The laboratory was open every school day from 9 a.m. to 3 p.m. except for all-school activities such as Field Day or a school-wide assembly program. At mid-year we met with the math-lab aides to evaluate and discuss our progress to that point. At that time many of their questions could be answered, and many of their ideas were shared.

We accumulated our materials for the mathematics laboratory from numerous sources. We found some in a lower-floor storeroom and others in various closets and on shelves where teachers put things away to be used later. We gathered all these things in the mathematics laboratory. We made geoboards, abacuses, and walking yardsticks. We prepared booklets of exercises using compasses and geo-boards. The only monetary expenditure that was made for the initial mathematics laboratory setup was the purchase of a commercially prepared set of tapes and the accompanying workbooks. The PTA supplied the funds for this. During the school year, individual parents donated various items to our laboratory.

The mathematics laboratory materials were divided into ten major categories, each coded by color.

1. General supplies: paper, pencils, erasers, grease pencils, graph paper, rubber bands, and so on.

2. Audiovisual materials: kits, film-strips, tapes and records.

3. Books: textbooks and bocks dealing with mathematical subjects.

4. Measuring devices: scales, balance beams, rulers, walking yardsticks, representations of cubic units, containers for measuring volumes, and meter sticks.

5. Geometric devices: geoboards, geoblocks, algebraic geoboards, protractors, compasses, and exercise booklets.

6. Graphing materials: graphing kits, activity booklets and maps.

7. Computing devices: adding and calculating machines, slide rules and Napier's Rods.

8. Concrete representations of fractions, decimals and percents: magnetic fractional parts, fractional parts kits and conversion devices.

9. Counting devices: abacuses, counting frames, calcuframe, Cuisenaire Rods, multi-base abacus, chips and dice.

10. Games and puzzles.

In putting our mathematics laboratory into operation, we are using three phases: Phase I--remedial; Phase II--accelerated, and Phase III--average. We selected

students to participate in the first phase of the program through the use of diagnostic tests and teacher recommendations. We began with upper-grade students, especially those who were definitely lacking in basic mathematics skills. In the first year of the program, one-fifth of the fifth and sixth graders and a few fourth graders received specialized help with their particular problems. Each child who was assigned to the program spent at least one-half hour per week in the laboratory; some worked in small groups of two or three, others worked individually. The laboratory schedule was arranged so that a math-lab aide would be working with the same children each time he came. In the beginning the teacher prescribed the course of action to be pursued during the laboratory period. Later, as the aides bacame more proficient, they branched out on their own, with the teacher's approval. The math-lab aides recorded what each child had done during his visit to the laboratory on the individual student's Progress Sheet. Thus, the essential two-way communication between the teacher and the math-lab aide made it possible to provide a beneficial and specific program for each individual child.

In Phase II of the program, a table-top computer, which was purchased with NDEA Title III funds as part of a district experimental program, will be placed in the laboratory. Training in the use of the computer will be provided for the mathematics teachers and for two of the math-lab aides. We hope that some of the more accelerated students will eventually do the programming for the less-able students, and in that way we shall be using the computer as a teaching device for both groups.

The reactions to our program have been overwhelmingly favorable. The results have proven it to be very worthwhile. Every child still attends his regular mathematics class. It has been interesting and quite revealing to observe the students who have participated in the laboratory program. In their regular mathematics classes, their improved attitudes and interest in mathematics have been quite noticeable. Children who had rarely participated in class were now eagerly waving their hands wanting to contribute. As one of the math-lab aides commented, "The enthusiasm is the thing I can't believe." Self-confidence has zoomed. In terms of achievement and test scores, we have seen a marked increase in the growth of the mathematics-laboratory students as compared to others who have not participated in the program. The increase is especially evident in the areas of concepts and concept applications.

Our math-lab aides were quite enthusiastic about the program, and many have asked to continue in the second year. The laboratory has proven to be of such interest that a number of teachers and administrators who have visited the laboratory have returned to their own schools to experiment with something of a similar nature.

We have been fortunate in having four ingredients that we feel are essential to the success of this program: (1) a willing administration that allows experimentation, (2) a willingness on the part of the faculty and staff to make the adjustments necessary to allow for some flexibility in scheduling students for the mathematics laboratory for special work, (3) parents who are willing to volunteer time as math-lab aides, and (4) students who are willing to make some sacrifices in order to receive special help. Some students even asked to work in the mathematics laboratory during their lunch-hour playtime in order to participate when they could not be scheduled during classtime.

We did not succeed with every student who was placed in the mathematics laboratory, but the growth that we have seen in the majority has made us feel that all our efforts have been worthwhile. We learned much in our first full year of operation. Two or three years from now our mathematics laboratory may bear no resemblance to what we have now. It is a process of continual change--one idea leading to another, one success or failure leading to adaptations of programs and methods. Our math-lab aides have contributed much in the way of helpful ideas and suggestions for improvement. We can see the potential for expansion. Several primary teachers have been considering the idea of adapting this approach to the needs of the children at that level.

We are looking forward to the future and the time when we can make the mathematics laboratory experience a reality for every child.

Criteria for "Hands-On-Learner-Orientation" in Mathematics

A. Learner Enviroment must be "Problem-Based":
1. Solves questions by means of something other then ROTE.
2. Problems are "events which have an element or degree of uncertainty or unknown." The learner "comes to" (or is brought to) the experiences of assessing and interpreting his uncertainty.
3. "Guesses" of either separate learners or learners in conjunction with others in a group should be encouraged and supported.
4. Approaches used by learners should have "a quality of generalizableness to the process utilized by the learners"; that is the learner is using an approach or strategy which could be usable in other problem situations.
5. Learner enviroment should cause the learner to ask questions which have answers of "indefinite character"; that is, there may be many possible answers, but NOT necessarily one single definite answer. In fact, a "good guess" may be the best possible answer or most "practical" answer.
6. The wording or stimulus situation used to describe a problem, needs to provoke learners to think. (Such words or directives as "always, never, usually, sometimes", as well as the "provacative questions -- Where does one begin? What is to be done? Are patterns begun that ask to be completed or extended? Where is it leading?
7. Fact-recall or memory (ROTE) of facts and information is utilized only to being the process or to try out certain hunches or possibilities during the process. (To memorize number facts, terms, or symbols is, of little use and often blatently ignores many basic learning principles.
8. An "element of discovery" ("Ah ha!" or "Erueka!") should be evident in learner's activity as he "solves problems."
9. Problem situations should be somewhat "open-ended" in that different learners may carry their thinking to different levels and by different routes.

B. Learner Enviroment must be "inquiry based": (Some thoughtto teaching strategies.)
1. Learners will have the major responsibility for the course of their own inquiry. (Teacher: "I will out wait them!")
2. Learners will have maximum freedom to determine what directions their investigations will take.
(Teacher "I will not approve or disapprove!" "I will serve as a source of data." "I will not tell them they are close to the RIGHT answers!")
3. Learners are not locking for a single answer, but are exploring for the purpose of building more useful "approached" and "theories" of "How to Learn to approach a problem situation".
(Teacher: "I will be open-ended and ask them to be also". "I Will not supply closure!" "I willemphasize that no inquiry is every really finished.")
4. Learners will not be given standards to meet.
(Teacher: "I will listen to their questions and not reject them, but will try to help them gather data, make inferences, form hypotheses, etc...")
5. Learners will actively attempt problem solving inquiry.
(Teacher: "I will not talk about inquiry into problems but allow them to conduct some inquiry -- hands on!")
6. Learners develop or originate their own theory about some problem situation.
(Teacher: "I will ask them -- What is a theory? Where do theories come from? What do you do with them?!")

Criteria for "Hands-On-Learner-Orientation" in Mathematics

C. YOUR TASK and MINE:

1. EXPLORE, MESS-AROUND, DABBLE, REACH-OUT -- GET YOUR HANDS ON (AND YOUR MIND OCCUPIED WITH) ideas and materials which will help you explore concepts and generalizations in math.
2. If you can develop and apply some of these guidelines "as a learner in this and other modules," there is a good chance that you will want to and will try to translate these guidelines into your work with other learners.
3. Generate many of your "hands-on attempts" with others. Separate-learner-attempts often do not produce much variety or depth of ideas or concepts.
4. Critique materials, aids, and procedures in terms of the above guidelines. If a material doesn't fit -- modify it, twist it, or "ignore it"!

PROBLEM-SOLVING ACTIVITIES OBSERVED IN BRITISH PRIMARY SCHOOLS*

Rose Grossman

The object of a trip to England was to visit schools that were actively involved in teaching innovative primary school mathematics programs. We met with Dr. Geoffrey Matthews, Director of the Nuffield Mathematics Project, who had worked with the Ministry of Education in planning visits to schools in various parts of greater London, Bristol, and Cambridge, including schools in high, medium, and low socio-economic areas, visits to Teachers Centers, and a visit to a teacher training college.

Conferences during the noon hour were used to exchange views, to ask and answer questions, and to discuss in some depth what we had observed in the morning classes. These luncheons were attended by the head teacher (the equivalent of our elementary school principal), several teachers, and occasionally by the local inspector (the supervisor in charge of a district).

Frequently we had further conversations during the tea hour and in the evening.

On some days we visited Teacher Centers, where infant school and junior school teachers came after school hours to become more familiar with math lab materials and to work with them under the guidance of a seminar leader (a previously trained teacher). These sessions were preceded by "tea" to encourage the teachers to come in unpleasant weather to an area often out of the way; and also, it was pointed out, to provide a social setting for casual conversation, during which teachers could compare notes, share enthusiasms, or give vent to their frustrations. This turned out to be an ideal time for the seminar leader to discover what conditions, both actual and attitudinal, existed in the schools.

A full day at a teacher training college impressed upon us the high correlation between the work being emphasized in college classes for teacher trainees and what was actually happening in the primary schools we visited. A great many physical materials are used in teacher training classes, with the emphasis on independent exploration and discovery by the college students. Prospective teachers begin their visitation in the primary schools with one week in their freshman year and continue visitation throughout their college stay for

*Reprinted from The Arithmetic Teacher, Volume 16, January, 1969, pp. 34-38.

increasingly lengthy periods of time.

Following is a decription of some of the highlights of our visit:

USE OF PHYSICAL MATERIALS:

An enormous amount of material, both commercial and homemade, was freely available to all the children in the room where the daily math program was taking place, whether it was in their classroom or in a laboratory center (the latter being located in the cafeteria, the gymnasium, a converted storeroom, or wherever space was available). Both materials and children often overflowed into the hallways.

The materials included multibase blocks, trundle wheels, color rods, Unifix cubes, learner's rulers, desk calculators, abaci, thread sculpture, geoboards, histogram boards, plastic shapes and containers of various sizes, dozens of little plastic boats, cars, tanks, and planes in a variety of colors (for sorting and grouping), ruler racks (twelve rulers of length 1 inch through 12 inches, to be used singly or in combination for estimating and measuring), measuring pole (for children's height), geostrips, clocks, scales, floor tiles, sand, plenty of water, etc.

The thing that impressed us about the material, beyond the tremendous variety, was the variety within the variety. For example, besides the sets of multibase blocks, there were other far less expensive commercial and homemade materials for exploring different number bases; there were at least three different varieties of geoboards of several different sizes; a Chinese abacus, a Japanese abacus, and one that could be converted to either; scales with weights, balance scales, equalizers of several kinds, and other materials.

There was no question as to the children's familiarity with or enjoyment of the materials. They answered our questions when we asked them, but otherwise were so absorbed in what they were doing that they hardly noticed we were there. As for discipline problems, there just weren't any, even in under-privileged areas.

We asked one of the teachers how her little ones of five, and six could find materials so readily and could follow their assignments (which necessarily required a minimum of reading). She indicated that during the first week or so, they had done no "work" but had gone through the motions of getting and discussing what they would do with these materials. She considered this a necessary prerequisite for the success of a program which called for so much mobility and independence on the part of the children.

GUIDEBOOKS AND ACTIVITY CARDS:

By the third day of visiting schools, a pattern began to emerge. It became apparent that the children were working chiefly with activity cards in conjunction with physical materials and were recording their responses in individual notebooks.

Where were their textbooks and workbooks? More important, where were these activity cards coming from? Who was making them? Where did they get the ideas?

PROBLEM-SOLVING ACTIVITIES OBSERVED IN BRITISH PRIMARY SCHOOLS

We raised these questions with teachers and head teachers of the various schools we visited and realized, from their replies, that we were onto a key aspect of the "revolution" that is going on in English primary education, that of individualization and nongradedness. Specifically, we discovered that these activity cards were an outgrowth of ideas to be found in the Nuffield guidebooks and Mathematics in Primary Schools.

Some resourceful teachers and head teachers were cutting up the better mathematics booklets to serve as activity cards. These paperback booklets replace textbooks and are to a large extent topically arranged with sections on topics such as symmetry, shapes, patterns of numbers, estimation, using the abacus, linear measurement, weight, dry measures, and graphs.

Although the activity cards had no distinct format (since different topics lent themselves to different development), the general plan seemed to be for the child to use certain materials in order to work out "problems," and to keep a record in his own notebook. A place for ingenuity or originality on the part of the child was built somewhere into the lesson. A follow-up card might have pictorial representations of the physical materials with "problems" to work out, and then the child would be asked to make up some questions of his own, based on his discoveries, for someone else in his group to work out.

On some cards, the children were asked to "guess" and then check their guesses with physical materials. Sometimes the checking itself required ingenuity as, for example, in (4) below. This activity card started rather simply with the following directions:

1. Without using a ruler draw a line which you think is 1 inch long. (If you cannot make the line straight, use the edge of a book to help you.) Now measure the line to see how accurate you were.

2. Draw lines, without using a ruler, as long as: 4 in., 2 in., 6 in., 5 in., 3 in. Now check your lines with a ruler.

3. Keep a record in your notebook, using these headings: My guess, Actual measure, Difference between "my guess" and "measure."

4. Now draw a curve which you think is 5 inches long. How will you check it? Do so, and say how you measured the curve. Add this to your record.

5. Repeat for curves as long as: 9 in., 11 in., 7 in.

This card was taken from L. G. W. Sealey's Facts to Discover and Learn and is reproduced only in part here.

Each child worked on an activity card either with a partner or within a group of three or four children. In many instances they worked independently, making use of the partner or group to talk things over when they needed to.

PROBLEM-SOLVING ACTIVITIES OBSERVED IN BRITISH PRIMARY SCHOOLS

ROLE OF THE TEACHER:

The appearance of the classroom in England is changing, even in old buildings where chairs and desks are stationary. More and more the classrooms give the "feel" of a studio or workshop, filled as they are to bursting with materials, displays on walls and screens and areas where children work in small groups, each group independent of one another. The room is filled with the bustle of movement and the hum of voices.

Concomitant with this change in the classroom is a change in the role of the teacher. She is no longer the authority figure, standing in front of the room, all eyes on her as she asks factual questions to which there is only one answer and, all too often, only one "approved" way of finding that answer.

There are necessarily occasions when the teacher addresses the class as a whole, but even this is done in a new way, because the thrust of her questioning is to emphasize "process"; that is, how the child is approaching the problem. The teacher is doing far more listening than talking.

Mostly, however, the children are in small groups and the teacher, mingling with the children, goes from group to group, asking individual children questions like, "How are you working this out?" "Can you think of another way to do it?" or "What would happen if....?"

It is obvious that the children are bing stimulated to be imaginative and independent. It is equally obvious that the teachers are, too.

One frequently heard the phrase "environmental mathematics." The teachers were finding a great amount of mathematics in the use of familiar materials such as acorns, leaves, floor tiles, empty cereal and candy cartons of various shapes, the flagpole in the playground, the classroom and the hallways, as well as in the physical surroundings of the community itself and the current events that touched on the lives of the children.

In one school, situated near the harbor, the children graphed the comings and goings of certain ships, noticed patterns, drew conclusions, and made predictions. From this, they went on to a project on general transportation. In another school, six- and seven-year-olds followed the progress of an apartment house that was being built, made graphs and physical models of it, speculated on how long it would take, based on data that came from an interview with the contractor, discussed even and odd numbers in counting the windows, which were in pairs, etc. In still another, children in the infant school were discussing angles, using the bend in their elbows as models.

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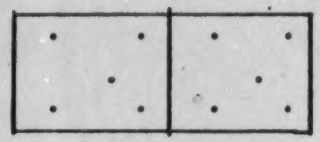
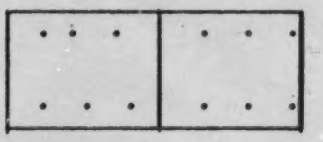
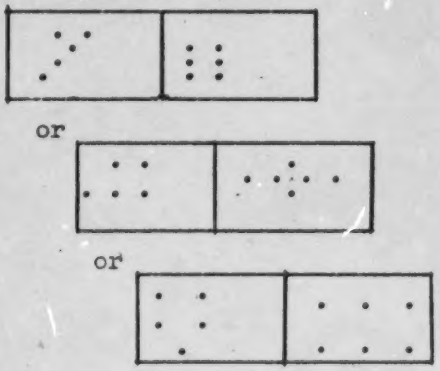
One teacher showed us a set of dominoes she had made for her class of six-year-olds. In the typical commercial set, each number is represented by dots arranged in a standard pattern. Thus, for example, "five" would always look like this:



and six always like this:



In her set (on cardboard and enlarged) each number was represented by dots arranged in a variety of ways. Five and six, for example, thus:



She pointed out that otherwise the children thought that only one pattern was right, and all others somehow wrong.

ROLE OF THE HEAD TEACHER:

One of the things that seemed very significant to me was the fact that, in addition to his administrative duties, the head teacher (the equivalent of our elementary school principal) teaches. Having presumably reached his position as head teacher because of his eminence as a teacher, he does not suddenly drop teaching and become solely an administrator, as all too often happens.

Every head teacher whose school we visited actually taught. By this I mean that he asked "process" questions just as the teachers did (i.e., "How are you going about it?" "Have you noticed anything special?"...) as he circulated among the children, and he was available to the children as a "consultant." He shared the teachers' enthusiasm and their problems. Being equally involved, he was as eager to have necessary materials as were the teachers, and so he found the money in the budget for it. It was made clear to us (with a certain amount of pride) that teacher's meetings dealt not only with administrative matters but,

to a great extent with sharing ideas, evaluating materials, books, and activity cards, and with preparing additional cards, as the needs of their own students required them.

I mention the above because it somehow helps explain the spirit of enthusiasm that pervaded the schools we visited.

COLLABORATION:

It is significant that there was a great amount of collaboration among representatives of different disciplines in the planning and preparation of the new primary mathematics programs of which the Nuffield Project is a striking example.

Not only were mathematicians involved, but teachers, nursery school people, psychologists (particularly of the Piaget school) educators in the teacher training colleges, as well as inspectors of various school districts. The name of Miss Edith Biggs, one of Her Majesty's Inspectors, came up frequently and appreciatively during out visits.

As a result of this collaboration, emphasis has been placed on the attitudes of teacher and children, on how to release children's creativity, on how children learn, and on the art of asking process-directed questions.

Another aspect of teamwork became apparent during out visit. British manufacturers are providing the physical materials (which they refer to as "apparatus") in great supply and variety, far beyond what is at present happening here in the United States. In fact, we are importing much of our mathematics laboratory equipment from England. Publishers, more and more, are providing the newer topically arranged paper-back booklets which call for the use of these materials and which are now the basis for acquiring or creating activity cards.

FAMILY GROUPING:

Family grouping was being used in an underprivileged area that we visited. The head teacher was very keen on it and discussed it with us in some detail. A five-year-old, entering school for the first time, was put in a class with six- and seven-year-olds, which often meant that he was in the same class as an older brother or sister. This gave him a sense of security and helped him adjust.

Other purposes were served as well. Five- or six-year-olds, exposed to the activities of a seven-year-old, often "caught on" and went ahead more quickly as a result. A slow seven-year-old might be grouped with a bright six without his feeling conspicuous. A five-year-old could ask a question of an older child (a shy child might even turn to his own brother or sister), not necessarily seeing the teacher as the only "authority."

PROBLEM-SOLVING ACTIVITIES OBSERVED IN BRITISH PRIMARY SCHOOLS

OTHER INNOVATIVE IDEAS:

It is not possible, within the scope of one article, to discuss all that we observed during our visit. We were able to get a glimpse of some of the great ideas that are developing. We were delighted to see persons at every level involved in the creative process-the head-teacher level, the teacher level, and the child level. Innovation has not been left solely to the original innovators but has been built into the program as an ongoing part of it.

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Why Manipulative Materials? *

Why manipulative materials? Because everyone touches and plays with objects--especially kids. "Don't fidget with the pencil." "Sit still." "Why are you rolling that piece of chalk down your desk?"

Frequently, manipulation of objects is essential to learning. My daughter can't learn to walk without manipulating her feet, arms and whole body. You can't learn to drive a car without first playing with the machine. It's necessary even if, like me, you agree there's much more to driving! And we can't learn to cook, play the piano or write without moving materials.

Moreover, it's fun to manipulate materials--aesthetically pleasing. That's what pottery, sculpture and yo-yo's are all about. Maybe this even extends to throwing spitballs!

Sometimes one manipulates materials almost unconsciously while one meditates (saying a rosary), concentrates on the end product rather than the manipulation (combing your hair), or thinks (doodling with a pencil, coat button or cigarette). Looking intently through binoculars one plays with the adjustments, and while chatting with friends most people pat arms, toss change in their pockets or scratch their chins.

It is natural and delightful for all of us to manipulate materials. In fact, it's hardto imagine life without doing it. Sometimes these actions are essential to learning. Can such behavior contribute to the learning of mathematics?

Well, clearly, some materials have always been used for this purpose. Fingers and counters, for instance. Unfortunately, children soon get a feeling that they shouldn't depend on such physical aids, but many teachers do not help them develop criteria for learning so that they can internalize a counting method and abandon their fingers.

New important aids have been produced over the past twenty years and are available to all. Colored rods, geometry boards ("geo-boards"), attribute blocks, geometry models, squares and cubes, and a variety of children's building kits are some of the ones being used. The materials are combined with activities led by teachers who understand their use. Many students learn from an awareness of what they are doing far more richly, joyously and efficiently than from spoken or written words and explanations.

Speech can be a very weak communicator of basic mathematical principles and memory of a second-hand, vicarious experience is a poor substitute for first-hand involvement. Why should children just watch the teacher become involved? Why can't they, too, be turned on by their own involvement? We try to achieve this in art, football, science, or singing. Children make music, throw pots, take part in discussion groups, cook, read for themselves and draw their own pictures. Why shouldn't they also participate in mathematics?

The aids mentioned above have important qualities over and beyond other materials. They are simple, relatively cheap to buy or make and easily used in classrooms by students of all ages.

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* From Cuisénaire Reporter, Spring 1972, pp. 3-4.

Why Manipulative Materials?

They lead--under the appropriate guidance--to an immense number of concepts, algebraically, arithmetically, and geometrically. After using them for eighteen years I still find some relevant fresh use. (Last week's discovery: a display of colored rods arranged in a fan with rods the length of your fingers, and another train of rods, representing the span of your hand. Not profound, perhaps, but potentially promising!)

The most important aids are multivalent materials. Their whole purpose is to act as bridges of communication. Each student manipulates materials and produces something. He looks carefully at what he has done. He talks about it and listens to accounts by class-mates engaged in similar work. The teacher probably adds useful conventional words and symbols which help students talk about it all and record their findings. But the teacher is almost powerless to add anything else--certainly not concepts. These have to be triggered within each person and do not come across from others. Together, students and teachers advance out of a corporate mixed ignorance toward understandings which, have no fear, are in accord with established knowledge and local curriculums. The teacher is not the only authority. A person is his own authority--he is the author of his own experience. We need children and adults who experience mathematics at first hand. When they do, they love it and they "get it"!

Do we use these aids all the time? Of course not! There is paper and pencil, talk and chalk, books, tables, many other things to use. A blend arising from a proper understanding of processes and facts is the pedagogic objective. Used inappropriately as magical substitutes for ignorance, insensitivity and alternates to other math programs, manipulative materials can be only dead wood. Far better to have left the trees alive and growing and stayed at home ourselves!

THE ROLE OF DISCOVERY: MATH *

The "discovery approach" is commonly described as a method of teaching by which the child solves problems by his own means. In such a situation, the teacher engages the child actively in the learning process, he provides the pupil with materials that stir his curiosity, and he encourages the child to find patterns, acquire concepts, and discover relations without a great deal of outside help. The atmosphere in a classroom where this approach is used must be free of tension so that the child is not afraid to make a mistake.

Though the general description of the discovery approach given above will probably not find many serious opponents, answers to specific questions such as the following often differ with different people:

What do we mean by "exposition," "discovery," and "guided discovery?"
For what purposes should the discovery method be used?
Should discovery of mathematical concepts and relations by the child be well-planned?
Can the child discover most of the mathematics learnings by himself?
After the child has acquired a concept, should it always be expected that he can verbalize the concept?
Is learning by discovery the most effective learning procedure for all teaching objectives?

The typical classroom teacher does not hesitate to state that not much pure discovery takes place during the mathematics lesson. Ability limitations and the time element involved lead to the teacher's guidance being required. Therefore, the process is commonly one of directed discovery. In such a situation the teacher's pertinent guide questions stimulate the child's thinking and his interpretations assist him in his building and refining of mathematical concepts, in his development of thought patterns, and in his seeing through relationships. In such discovery teaching, the pupil is stimulated to think critically, to find, use, and test various methods of solution, and to prove his conclusions. Not only the desire to arrive at a correct answer is fostered, but also a questioning attitude toward and a critical evaluation of procedures are encouraged. It is anticipated that such a method of teaching will stimulate curiosity, elicit internal motivation, and result in learnings and skills that are longer retained than those acquired by the method of exposition.

The extent to which discovery is used by the child in a learning process in the classroom appears to depend on various factors, including:

The child: Because of differing levels of ability, one child needs more guidance than another.

The teacher: The philosophy, attitude, and ability of teachers differ. Is the teacher a firm believer in the discovery approach? Does he like mathematics? Does he utilize all available opportunities to have the child develop himself? Does he provide suitable materials? Is he skillful in asking pertinent questions? Does he judge the child only by the final answer? Is the classroom climate free of tension?

The subject matter: A child may discover a mathematical concept such as the commutative property without much help, whereas he may be completely dependent upon the teacher's guidance to understand a process, for example, to see through the meaning of inversion when dividing by a fraction.

* From: Kramer: Problems in Teaching of Elementary Math, Allyn and Bacon, 1970, pp. 54-56.

The Role of Discovery: Math

The time element: The most ardent advocate of the discovery approach will at times be compelled to rely heavily upon exposition in order to complete necessary learnings within a reasonable time limit.

The teaching load: The number of pupils in the classroom may place restrictions on the teacher in his use of the discovery approach. As the number of students increases, the time that is available for individual pupil decreases.

The grade level: The extent to which discovery can be used appears to depend upon the amount of knowledge that has been acquired by the student. It seems logical to assume that the typical sixth grader needs less direction than the typical kindergartener.

The availability of equipment and materials: Discovery is encouraged when the school provides a mathematics laboratory where pupils can engage in projects, make models, and work with such materials as mathematical games, puzzles, devices, and calculators.

ASKING CHALLENGING QUESTIONS

To teach arithmetic effectively is to ask children challenging questions that dig toward the heart of the subject.

In that statement we mean to distinguish between problems that require considerable thought and obvious questions that tax only the memory. We firmly believe that we can help children learn to think if we stimulate much more than memory of facts and procedural steps.

Math Workshop for Children* has its sights set high. One of its goals is to develop facility with the basic facts of arithmetic so that children will not stutter and stumble painfully in this important subject. But its central purpose is to help teach children to think.

The technique for both is the same: to ask challenging questions which require children to use many facts of arithmetic as they work toward solutions.

There is nothing really new in this method. It was used effectively and developed into an art by Socrates, one of the greatest teachers of all time. It only seems new and strange to many of us because it has been preserved by so few people and encouraged by only a few writers of instruction materials in arithmetic.

Parents as well as teachers can help children most skillfully as they develop the ability to ask challenging questions or to recognize when the text materials ask such questions— and then encourage children to find reasonable solutions.

What Is A Challenging Question?

If a question is to be useful, it must be clearly understood by the children. If it is not an obvious question, it requires creative thought. If it is to be effective in the learning process, its answer must not be beyond the children's level of development.

As Dr. David Page of the University of Illinois Arithmetic Project has said: "Anyone can ask a question so easy almost everyone can answer it without effort, or so hard almost no one can answer it regardless of how hard he tries; but our task, if we are to help children, is to pose questions to which children can find answers by calling up from memory what they have learned and by rearranging that information in a way that helps solve the problem at hand."

Asking challenging questions is a difficult task for many teachers and parents. One of the primary purposes of Math Workshop for Children is to ask many such questions and suggest others.

Each page, as a whole, presents such challenging questions as we are talking about. Where does one begin? What is to be done on this page? Are clues given? Are patterns begun that ask to be completed or extended? Such pages are not familiar to those whose backgrounds have been limited to lists of obvious questions that merely test the memory. If we are to help childfen learn to think, we must, indeed, include these obvious questions, but in a larger activity. Creative teaching presents them as patterns that lead somewhere, as stepping stones to new problems.

Asking Challenging Questions

An Example of a Challenging Question:

Here are nine instances of drill presented in the traditional format. Each asks a question in the shorthand of arithmetic:

$3 + 0 =$ _____
$2 + 2 =$ _____
$1 + 3 =$ _____
$4 + 0 =$ _____
$2 + 1 =$ _____

$$\begin{array}{r} 0 \\ +\,4 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ +\,2 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ +\,3 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +\,1 \\ \hline \end{array}$$

If children can answer these questions, then the activity constitutes drill— and that's all. Practice in recalling these facts is, of course, essential. But in the exercises given above, the sights are set far too low.

However, we can include those nine instances of drill as an integral part of finding the answer to a much more challenging question. And because the mind is alert and searching, it is receptive— it will remember more efficiently the steps it moves through.

Question: Fill in as many of the following blanks as you can, so that each completed sentence of arithmetic is true (using only the whole numbers of counting and zero):

_____ + _____ = 3
_____ + _____ = 3
_____ + _____ = 3
_____ + _____ = 3
_____ + _____ = 3
_____ + _____ = 3

_____ + _____ = 4
_____ + _____ = 4
_____ + _____ = 4
_____ + _____ = 4
_____ + _____ = 4
_____ + _____ = 4

This activity involves exactly the same drill items listed earlier in traditional format but new elements have been introduced.

Are there six different ways of finding pairs of addends that equal 3 and 4 in the form of _____ + _____? That is a bit of a puzzle— a challenge.

As children call up and record the basic facts involved, they meet a perplexing situation. After finding some other names (pairs of addends) for 3 and some other names for 5, what next? Are there more? Can you be sure that you have found them all?

Suppose the facts are listed in the following order:

$0 + 3 = 3$
$1 + 2 = 3$
$2 + 1 = 3$
$3 + 0 = 3$

$0 + 4 = 4$
$1 + 3 = 4$
$2 + 2 = 4$
$3 + 1 = 4$
$4 + 0 = 4$

This is a convincing argument that all possiblities have been included.

Other questions are in order. How many of the following blanks can be filled in as different names for 0, 1, and 2?

Asking Challenging Questions

____ + ____ = 0
____ + ____ = 0
____ + ____ = 0

____ + ____ = 1
____ + ____ = 1
____ + ____ = 1

____ + ____ = 2
____ + ____ = 2
____ + ____ = 2

All the results thus far are worth summarizing:

0 + 0 = 0 0 + 1 = 1 0 + 2 = 2
1 + 0 = 1 1 + 1 = 2
2 + 0 = 2

0 + 3 = 3 0 + 4 = 4
1 + 2 = 3 1 + 3 = 4
2 + 1 = 3 2 + 2 = 4
3 + 0 = 3 3 + 1 = 4
4 + 0 = 4

Can you guess how many names we can find for 5 in the form ____ + ____ = 5? How many names for 6 in the form ____ + ____ = 6? Some might like to guess how many names can be written for 12 in the form ____ + ____ = 12 and then test his guess by making a list.

Compare the two techniques outlined above that start by asking essentially the same questions but in different forms. The first, or traditional, listing of nine obvious questions is a dead-end testing of the memory. However, when a challenging question is asked, the use of memory is only the beginning. A puzzle aspect enters New questions arise as the children work ahead. Patterns are developed. An interesting and unexpected conclusion looms up ahead. The more able children will form hunches about some big ideas and test these hunches, each working at full capacity.

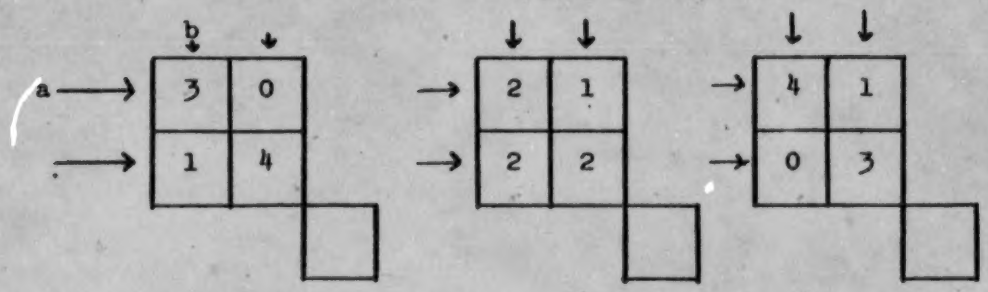
Working toward the solution of such questions, children's minds are alerted, they encounter large doses of fact-recall, they meet surprising developments, they have the thrill of discovery, and feel pride in achievement. The resulting activity is open-ended, and each child is called upon to work to his highest level.

Another Example of a Challenging Question:

In a different way, we will ask another question, and begin with the same items listed above in the form of a traditional drill exercise.

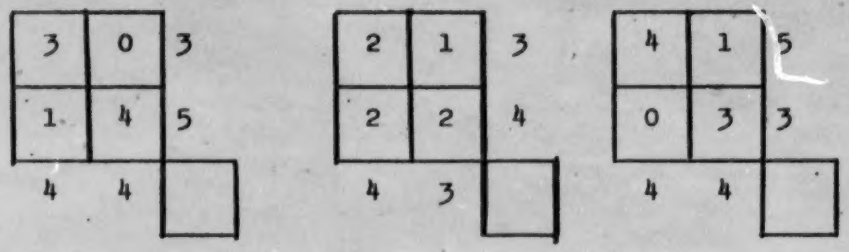
In the little diagrams below we explain that each row and each column should be read from left to right and from top to bottom respectively. The top row of the first example (marked a) should be read 3 + 0 = _____, and the result written on the right. The left-hand column in the same example (marked b) is to be read as 3 + 1 = _____, and the result then written at the bottom of that column.

Asking Challenging Questions



To complete the three examples requires twelve acts of recalling basic facts of addition. The nine listed in the traditional drill exercise are included; two others are introduced — 1 + 4 = _____ and 4 + 1 = _____; one is repeated — 2 + 2 = _____.

The results look like this:



This activity has a new look. At first glance, its purpose is not obvious, although it has certainly condensed a number of "obvious questions" into a small space.

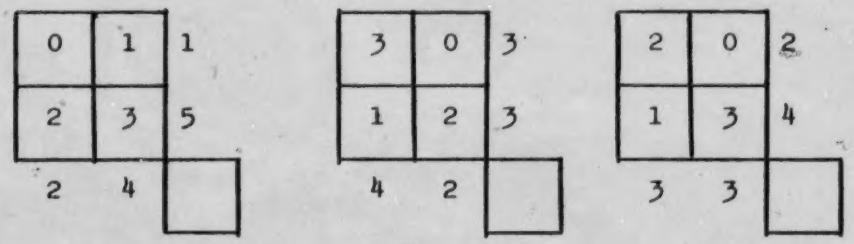
Where is it leading?

What is the purpose of the box in the lower right-hand corner? Children almost never need to be encouraged to wonder about this box. They have a strong feeling that something ought to be done with it.

However, the question about the purpose of the box is not obvious. After some discussion, it is noticed (perhaps only by the more proficient) that the sum of the two numbers listed above each box and the sum of the two numbers listed on the left of each box are the same: 8 in the first example, 7 in the second, and 8 in the third.

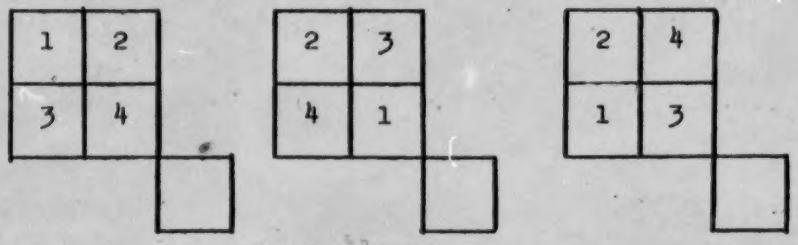
That is certainly a surprising development.

Let's try other examples, using different arrangements of 0, 1, 2, and 3 in the group of four boxes:



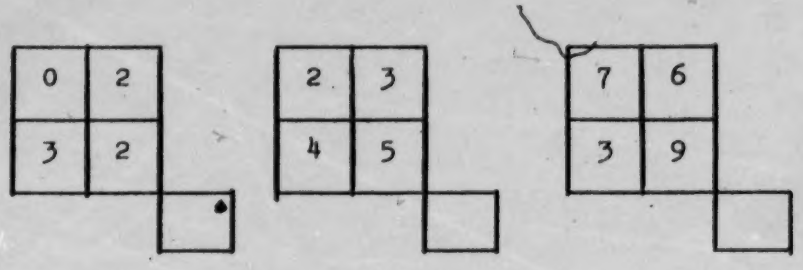
Another surprise! The bottom right-hand corner box can be used at it was in the original examples. But in these three, 6 will be written in each example!

Let's try arrangements of 1, 2, 3, and 4:



At this point, most children want to "try some" on their own. And the search is on for other groups of four numbers that "work out" in the same way.

Thus, mathematical research is under way in the first grade. Children are able to find some groups of four numbers that "behave" in this interesting fashion. The more able children search for examples in which larger numbers are used. The successful discoveries may span quite a range:



After considerable experimentation, some child in the class usually makes a guess that any four numbers will "work" in this same surprising way. This turns the search toward finding an example that will prove the guess is wrong. Since there is a suspicion that large numbers may not behave as well as small numbers, each child uses the largest numbers he can handle.

Each time a child finds what appears to be an exception, he discovers that the exception was due to an error in computation. This gives further support to the conjecture that any four numbers will "work" as well as any other four in this little mathematical structure we call the "Cross Number Puzzle."

Thus, a challenging question was posed. It required countless recalls of the basic facts, thus testing and reinforcing the memory; such practice was most effective because the mind was searching and therefore highly motivated. There was discovery, achievement, and a growing feeling that arithmetic can be an exciting and rewarding endeavor.

Furthermore, the children are on their way toward a firm conviction that the order in which a given group of numbers is added does not affect the outcome.

USING PUPIL TEAMS IN TEACHING*

Frank W. Lanning

We hear a great deal these days about "grouping for instruction." For the most part, its aim is to better fit our instruction to the individual pupil, and to restrict the inevitable competition between pupils to those of comparable abilities. Although the label we attach to this practice is grouping, our primary concern is still with the individual, and almost forgotten are the potentials of a group operating as a group.

Is there educational merit in having a group, or team, working together to solve arithmetic problems? Can the individual student profit in understanding and increase his personal achievement in arithmetic for having taken part in an arithmetic team effort?

It is the contention here, based on actual experience, that these questions can be answered with an emphatic "yes".

GETTING READY FOR TEAMWORK IN ARITHMETIC:

There are two important things to keep in mind as you attempt to group students for arithmetic teamwork. First, the grouping is not merely for classification purposes; its members will be working together as a team. Second, subsequent observations of each group at work will probably suggest adjustments, perhaps even a change in criteria for selecting group members.

One way - or partial - to choose your groups is to let the pupils decide. Have each pupil list on a piece of paper the names of three or more classmates he would like to work with. Record these choices in a sociograph, like the one shown in the figure. Simply list the names of your pupils in alphabetical order across the top of a sheet of paper and down the left-hand side. Indicate with a mark the names of the persons chosen by each pupil. By counting across you can find the total number of choices each person gave. You will also have a record of those students chosen by each pupil and those who chose each other. The information provided by this sociometric device can be of great assistance to you in identifying pupils who are more likely to work well together.

		A	B	C	D	E	Choices Received
Student Names	A		x	x		x	3
	B	x		x		x	3
	C		x			x	2
	D	x		x		x	3
	E	x			x		2
Choices Given		3	2	3	1	4	

USING PUPIL TEAMS IN TEACHING*

Frank W. Lanning

Groups may also be chosen on the basis of ability. Evidence of achievement levels from standardized tests and/or teacher-made tests in arithmetic may be used as the basis for establishing group membership. But remember, even though ability is your criterion for grouping, these groups are formed for the primary purpose of working together on problems in arithmetic.

Often the teacher's observations, his anecdotal records and general personal experience with the pupils in class represent as valid a criterion as any in deciding which pupils should form which groups. Teacher judgment usually correlates rather highly with tests and measures of pupil personality, motivation and achievement.

Very likely you will find there is no one best way to select your groups. Whatever techniques you do employ, you will still need to watch the groups in action and judge whether further admustments will need to be made.

Finally, a word about the size of the groups. So far, experience with this method has favored groups of five to seven pupils. This seems to be about the optimum size in which all or most of the members of a group can actively participate in teamwork solutions of arithmetic problems.

GETTING TEAMWORK OPERATIONS UNDER WAY:

Once you have organized your class in arithmetic "task forces" you have added another valuable means of instruction to your existing methods. You will probably find that teamwork activities are best interspersed throughout the year, either for enrichment activities or as an integral part of your program.

At the intermediate level, for instance, a teamwork project might be initiated by giving a set of story problems to each of the teams, with their goal to solve the problems in as many different ways as possible. At the primary level, the problems would be simpler, with perhaps the goal being to show solutions by means of concrete or semi-concrete materials.

At the outset, all teams would probably work on the same set of problems. As you observe the work, interest and capabilities of the various teams, subsequent problems would tend to differ as you attempt to adapt and make them more challenging to the members of the team.

Many arithmetic problems or projects can be found in other subject areas. From science; Constructing weather instruments such as the vane, barometer, wind-speed indicator, rain guage, in which dimensions need to be determined and measured, scales drawn, data recorded and averaged. From social studies: Outline maps involving scales, distances between cities, speeds, rates and times.

Although most of the teamwork problems and projects can be planned to be carried out within the normal arithmetic period, some will extend beyond. This is reasonable, for interest within some groups will certainly "spill over," and some enrichment projects are best worked on outside regular class time.

As to procedure, here are seven points that should be considered:

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1. Explain to the members of the groups that they are to arrive at an answer or answers by cooperating with one another. It will be very important that pupils in each group understand that they are to work together for a team-solution. Help them to understand that cooperation on the solution of an arithmetic problem does not mean that they are merely to copy from someone else in the group, or that they are to work independently and not share in the process of solution. Explain to them that cooperation does mean asking and answering questions of others about the methods used and how to solve the problems. Emphasize that cooperation does mean a direct comparison and challenging of each other's written work, or work done with other materials — blocks, sticks, cutout figures— used in the solution of arithmetic problems.

You may want to designate the chalkboard as a "challenge area." Here members of each team are free to show and explain their methods of working a problem, or perhaps to show why a particular solution is wrong.

2. Emphasize that all members of a group should be able to explain the procedure for obtaining the solution submitted by the group. Perhaps it is the ideal situation that would have all members of a given group able to explain how a particular team-problem was solved. Nevertheless, this goal should be set before the members of the group to help insure that each pupil will make some active effort to understand how a particular problem might be solved. Each group should submit a single paper representing what the group has decided as the best solution or solutions: also, work papers of the members of the group should be collected. Through this joint, single paper, stress is given to the fact that the group is engaged in a teamwork effort.

3. Rotate leadership responsibility among members of the group. Groupings of this type provide an excellent opportunity for many more pupils to share in the responsibility of being a group leader or chairman. Since the groups are small, rotating the leadership yields several important effects. For one, pupils tend to accept more readily the leadership of another group member because they know that they too, eventually, will be in the role of leader. For another, failure in the leadership role is generally not so critical— also less likely— because only a small part of the class (five to seven) is involved.

4. The time allotment should be sufficient. Experience with this kind of teamwork approach suggests that better results occur when the tasks for the groups can be initiated and completed in one block of time. There are exceptions to this, of course. Some very useful arithmetic projects may take several class periods. In either case, be sure to allow sufficient time for the groups to complete their tasks and thus provide for greater "feedback," i.e., immediate knowledge of results and the reinforcement obtained by completing the tasks.

5. Supervise the groups and be aware that groups have "individual" needs. Groups as well as individuals need your guidance. You will probably have to help the groups make use of all their members. Through your observations of the groups at work and your knowledge of their members, you will often be able to anticipate trouble areas or help a group better understand the nature of the problem and the process of sharing in the solutions.

Remember, groups have "individual" problems. Some groups will complete their

Using Pupil Teams In Teaching*

Frank W. Lanning

tasks quickly, while others will be slow or have trouble in working together as a group. The effectiveness of teamwork in large part depends upon how well the teacher can handle these and comparable problems.

6. Make use of discussion periods so that each group has a chance to tell or show other groups what they have done. Increasing the amount of discussion and the exchanging of ideas by the pupils within the group is important. To help make the group more effective as a team it is equally important that the small groups have chances to compare, show and discuss their problem solutions with the other groups.

If possible, and whether or not the task is completed, try to have the leader or chairman of each group make a brief progress report before each arithmetic period is over. This serves at least three useful purposes: It allows members of each group to compare their work and progress with that of other groups; it gives you, the teacher, some added opportunities to compare the groups with one another for more effective evaluation; it helps the pupils of each group to identify themselves more strongly as members of a team.

7. Use a variety of arithmetic problems. Three general sources of arithmetic problems are well suited for teamwork attack. They are:

a. Stated problems, usually involving two or more steps.
b. Activity projects involving actual measuring and weighing of materials, e.g., map work, scale drawings, construction of a simple star-sighting device (astrolabe) for an astronomy unit.
c. Arithmetic puzzles involving abstract reasoning.

Under these headings many kinds of lesson plans or problems can be developed and used for challenging the groups in a team-search for solutions.

Stated or story problems involving two or more steps help build and reinforce fundamental concepts. The story problem gives each pupil in the group a chance to read and to interpret for his teammates his understanding of the processes involved. Not only can the groups be active in solving story problems, but they can be encouraged to construct their own story problems, complete with solutions, to challenge other groups in their solution.

Arithmetic projects assigned to small groups differ from the usual work with textbook problems in at least two ways. First, there is generally a need for a longer, more comprehensive planning session on the part of the groups to organize the steps necessary to complete the project. Second, there is more opportunity to distribute leadership responsibility because of the usual variety of tasks needed to arrive at solutions.

For example, let us look at a project that requires each group to plan and construct a bird feeder.

There are questions of design: "What should be the dimensions?" "What kind of shape should it be?" There are questions of materials: "How much wood?" "How much paint?" "What size and how many nails?" "What will it cost?" These questions represent the kind of quantitative planning that the groups will need to do. Not only will completion of the project call for many ideas from all members of the group, there will be also a need for many skills, quantitative and mechanical. A project

of this sort should come close to utililizing much of the capacity of the group.

There are many other examples of projects such as map making, scale drawings, compiling data for use in constructing graphs and others that can make demands on both the student's qualitative and quantitative understanding of arithmetic.

The use of arithmetic or mathematical puzzles adds "spice" to problems to be solved by teamwork. There are many such interesting oddities in arithmetic and they are readily found collected in books, in supplemental material to some textbooks, in magazines and by word of mouth from the pupils themselves. Puzzles can help reinforce some of the more intriguing relationships and processes of arithmetic. Be it a magic square, finding an unknown factor or arranging some numbers or blocks to achieve an answer, puzzles are the "dessert" for the often solid "meal" of arithmetic.

EVALUATING TEAMWORK:

Evaluation is an important part of the teamwork method. A simple check list can be used while observing or helping the groups, and also at the time of the culminating activity of a group discussion or oral reports.

The following list suggests data of a kind that might be gathered:

1. Time for completion of problems (minutes).
2. Correct answers (number right and number wrong).
3. More than one method discovered and/or used for solution (yes or no).
4. Understanding of arithmetic concepts by the group (low, average, high).
5. Cooperation of group members (low, average, high).
6. Interest and satisfaction with group problem-solving (low, average, high).
7. Effectiveness of leadership (low, average, high).

A check list based on such items provides an objective record of your observations. It also allows for a comparison of observations made at different times. The data you include on your check list should be data that may be used directly in evaluating the work of the groups and, of course, should be consistent with the goals you set for this teamwork approach to arithmetic.

Written work of the groups should be evaluated, too. The group paper usually shows the agreed-upon procedures and solutions. The papers from the individuals are generally in the form of worksheets. These individual papers provide some indication of the extent of understanding and participation in the giving and receiving of aid in arriving at the solutions to the problems. Evaluation of both group and individual papers, then, should determine degree of understanding and suggest directions for additional improvement.

Comparison of the groups will undoubtedly arise. As we have said, groups, like individuals, show different characteristics. Some groups may be effective in teamwork for arithmetic; some may not. You, as a teacher, should be aware of these differences and try to seek answers for such questions as: "Why is Group A doing a superior job?" "Why wasn't Group B able to arrive at a solution?" "Does Group C need more direct help?" Why did Group D work so well with very little direction?"

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Comparing one group with another, seeking some of the reasons "why" and making adjustments when necessary is of crucial importance to your getting the most from the teamwork technique.

In summary the teamwork approach as a method of instruction can give you a chance to supplement individual problem-solving with group problem solving. Despite the laments of some, the fact remains: Increasingly, the mode of attack on the real problems of life are team attacks; more and more our problems are becoming far too complex to yield to the knowledge of the individual alone.

In the arithmetic class, this teamwork approach offers an opportunity for pupils to use their arithmetic vocabulary and understandings in meaningful situations. The method encourages pupils to ask of one another the "how", and especially the "why", of arithmetic. Finally, the technique demonstrates first hand the value of teamwork and the part it can play in the solution of many problems.

* From Updating Mathematics (1964), 65 - 68

Robert B. Davis

My remarks are based upon one view of the future needs of the mathematics curriculum in the elementary school. I want to indicate briefly what this view is and why it implies a need to bring a higher level of mathematical expertise inside the walls of the elementary school house.

First, what are the needs of the elementary curriculum in the next few years? I would argue that they are primarily these:

1. An early introduction of the "big ideas" of mathematics.
2. Replacing "arithmetic" with a combination of arithmetic, algebra, geometry, and science.
3. Developing a more student-centered approach to classroom instruction in mathematics.
4. A greater emphasis on student creativity and originality.
5. A more extensive use of physical materials in the mathematics classroom.
6. More emphasis on individualized instruction in mathematics.
7. Freeing the curriculum from its present bondage to grade levels and textbook series.
8. Teaching a great deal more mathematics in the elementary school than is ordinarily done at present.

These eight directions for change may require a word or two of explanation:

EARLY INTRODUCTION OF "BIG" IDEAS

Nearly all modern curriculum studies, in any subject matter field whatsoever, usually advocate identifying the central "structural" ideas of a subject and introducing them to quite young children. The argument in support of such a course of action is simple: over several thousand years man has developed effective "basic" ideas that organize a large body of knowledge in a powerful and efficient way. If we teach these "organizers" to young children, they will make use of them in all of their subsequent learning. If we do not teach these "organizing" ideas, then either the child must discover them for himself--which requires the genius of centuries of Euclids and Newtons and Cauchys and Hilberts--or else the child must get along without them and fight his way through a dense forest of "facts" with nothing by the way of a conceptual road map to help him maintain his bearings.

The "big" ideas of mathematics include such concepts as variable, open sentence, truth value of a statement, function, graph, the arithmetic of signed numbers, the relation of abstract models to physical reality, implication, contradiction, axioms and theorems, matrices, inference schemes, and so on. We know from various experimentation in recent years that all the ideas mentioned above can be understood by elementary school children.[1]

REPLACING "ARITHMETIC" WITH A COMBINATION OF ARITHMETIC, ALGEBRA, GEOMETRY, AND SCIENCE

This is not entirely different, of course, from the preceding point. The argument in support of such a "unified" or combined" program of study is primarily this: arithmetic, by itself, has little capacity for continuing growth. When, however, we combine it with algebra, geometry, and science, we have a curriculum that contains within itself the seeds for its own future growth and development. We expect the child to grow with each successive year; the teacher should also, and so should the school program. The school program at any grade level for 1967 should be a natural outgrowth of, and an improvement upon, the program at the same grade level that was taught in 1966. A unified program will tend to grow in this way, because it embodies a potential for growth. A straight "arithmetic" program does not embody such a potential for growth and will tend to stagnate. In a straight arithmetic program we would find our schools teaching the same things in 1970 that they taught in 1960, 1950 and even 1940.

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* From: Committee on the Undergraduate Program in Mathematics, Report Number 13, April, 1966, pp. 27-31.

DEVELOPING A MORE STUDENT-CENTERED APPROACH TO CLASSROOM INSTRUCTION

Much has been written lately about "discovery" teaching, about flexible sequencing of topics of study, and so on. At the core of all of this is probably the idea of interactive teaching, in which tomorrow's classroom discussion and study grow organically out of today's discussion and study. An example may help to show what this means. In a fifth and sixth grade "mixed" class in the Chicago Public Schools recently, some students made up the function indicated by the table

□	△
0	-3
1	-2
2	0
3	1
4	3
5	4
6	6
7	7
.	.
.	.
.	.

Other students in the class had the task of finding an algebraic formula to represent this function.

Now, in fact, the table above came about from an unintentional systematic arithmetical error that involved neglecting remainders after dividing by two. What kind of class discussion should now ensue?

The teacher was familiar with the mathematical function, "square brackets,"

$$[X] = \text{the largest integer } N \text{ such that } N \leq X,$$

and was also familiar with the creative work David Page has done in using this idea with young children. Hence, the class discussion was allowed to move in this direction, and the function in question was identified as either

$$\left[\frac{3 \times \square}{2}\right] - 3 = \triangle \qquad \text{or (if you prefer)} \qquad \left[\frac{\square}{2}\right] + \quad - 3 = \triangle$$

This led to considerable valuable work in estimation, "rounding off," and so on, as well as an example of a new type of function.

If the teacher is to allow class discussion to proceed organically as a continuing inquiry, the teacher must have a broad knowledge of the mathematical terraine that is being explored.

GREATER EMPHASIS ON STUDENT CREATIVITY AND ORIGINALITY

Studying tape-recording of many classroom lessons makes it clear that teachers are confronted almost daily with student suggestions that are very clear and deserve consideration. Most of these student suggestions are ignored or rejected by most teachers or else accepted uncritically in a fashion that leads to homogenized chaos. An actual example may help (from a Weston, Connecticut, third grade class):

The third grade teacher was explaining, "I can't take 8 from 4 . . ."

$$\begin{array}{r} 64 \\ -\ 28 \\ \hline \end{array}$$

So I regroup the 60 as 50 + 10 . . ."

Kye, a third-grade boy (who had learned the arithmetic of signed numbers in the second grade), interrupted: Oh, yes, you can! Four minus eight is negative four . . .

$$\begin{array}{r} 64 \\ -\ 28 \\ \hline -4 \end{array}$$

. . .and 20 from 60 is 40 . . .

$$\begin{array}{r} 64 \\ -\ 28 \\ \hline -4 \\ 40 \end{array}$$

. . .so the answer is 36 . . .

$$\begin{array}{r} 64 \\ -\ 28 \\ \hline -4 \\ 40 \\ \hline 36. \end{array}$$"

This particular teacher recognized the merit of Kye's method, and some of the children in the class went on to study it.

Kye was lucky. In many classes the teacher would have said, "No, Kye, that's not the way you do it. Now watch carefully, and I'll go through it again."

In such a "traditional" class, Kye would then have been left with an unresolved conflict between his own mental picture of how mathematics works out versus the authoritarian pronouncements of the teacher. For some children this might be a challenge, but for others it would be more than they could cope with.

There is no need here to continue this catalog of what is versus what might be. All of the proposed changes in the curriculum will tend to require more mathematical knowledge from elementary school teachers--in some cases, very much more. Rather than present an argument couched in the unconvincing rhetoric of educational English, I want to plead with the interested to view a few Madison Project films, showing actual classroom lessons. Is it not true that these children are enjoying themselves more than most children do in most schools? Is it not true that they are learning much more mathematics than children their age usually do? And is it not true that one indispensable step toward bringing this sort of thing to many more children is to educate more teachers who are flexible and creative, who conduct a student-centered classroom, and who are at home in the world of mathematics?

If so, our course is clear.

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[1]The best evidence I can cite in support of this claim is the collection of 16 mm. sound motion picture films made by the Madison Project that show actual classroom lessons with children, K-9. These films are available from: The Madison Project, 8356 Big Bend Blvd., Webster Groves, Missouri 63119.

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Clarence Ethel Hardgrove and Ben A. Sueltz

PRIMARY CLASSROOM

Number and Operation

1. Counters (cubes, buttons, tongue depressors)
2. Counting frames
3. Number cards and domino cards
4. Number chart (1 - 100)
5. Pocket place-value chart
6. Abacus (demonstration and individual)
7. Peg board (100 and 1000)
8. Number scale

Measurement

1. Length: inch squares of paper, flexible ruler made of inch squares, ruler calibrated only in inches , ruler calibrated only in half inches, ruler calibrated only in quarter inches, flexible yardstick made of foot length, flexible yardstick made of inch lengths.
2. Time: calendar, schoolroom clock, clock face with adjustable hands, mantle clock
3. Weight: simple teacher-made balance scales
4. Counting measures: egg cartons of different shapes, sections from egg crates
5. Money: play money (real money when available from school function)
6. Temperature: room thermometer, large adjustable model thermometer
7. Liquid measure: measuring cup; half-pint, pint, quart, half-gallon, and gallon milk containers
8. Dry measure: pint, quart, half-bushel, and bushel baskets
9. Geometric ideas: cardboard cut-outs of circles, triangles, squares, and rectangles; objects in shape of cube, cylinder, sphere, and cone

Relationship

1. Counters
2. Fractional cut-outs
3. Paper for folding into fractional parts

INTERMEDIATE CLASSROOM

Number and Operation

1. Counters
2. Place-value pocket chart
3. Abacus
4. Peg board (100 and 1000)
5. Number scales: whole number, common fraction, and decimal fraction

Measurement

Instruments for the measurement of

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* From: "Instructional Materials," Instruction in Arithmetic, Twenty-fifth Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1960, pp. 240-242.

1. Length: inch squares of paper, ruler calibrated in quarter inches, ruler calibrated in eighth inches, ruler calibrated in sixteenth inches, ruler calibrated in tenth inches, yardstock, odometer, tape measure, pedometer
2. Time: calendar, room clock, stop watch, sun dial, hourglass, timetables, standard time chart, time line
3. Weight: simple balance scales, nurses' scales, height-weight charts
4. Counting measures: egg cartons, sections from egg crates
5. Money: money from classroom functions, tokens, trading stamps
6. Temperature: classroom thermometer, clinical thermometer
7. Liquid measure: measuring spoons and cups: half-pint, pint, quart, half-gallon, and gallon milk containers
8. Dry measure: half-pint, pint, quart, half-bushel, and bushel baskets
9. Surface measure: inch square, foot square, and yard square cut-outs from newsprint; cross-ruled paper
10. Solid measure: cubic-inch wood cubes (more than 144), material for construction of a cubic foot and cubic yard
11. Geometric ideas: cardboard cut-outs of circle, triangle, square, and rectangle; objects in form of cube, sphere, cylinder, cone, rectangular prism

Relationship

1. Counters
2. Parts of egg cartons
3. Fractional cut -outs
4. Paper for folding and cutting (8½ x 11" used notebook or typing paper cut in 8½" x 2" strips)
5. Tray of 100 blocks or chart of 100 squares
6. Fraction board
7. Number scale: whole number, common fraction, and decimal fraction
8. Materials using fractions: ruler, liquid measures, odometer, pedometer
9. Maps, diagrams, and other scale drawings
10. Tables of values from texts, newspapers, etc.
11. Graphs (bar, line, circle, and pictograph) from magazines, texts, etc.

AN ANNOTATED BIBLIOGRAPHY OF SUGGESTED MANIPULATIVE DEVICES

Patricia S. Davidson**

This bibliography of manipulative materials has been organized into fifteen general categories: Blocks, Calculators/Computers, Cards, Construction, Drawing Tools, Geogoards, Measuring Devices, Miscellaneous Items, Models, Numerical Games, Puzzles, Shapes and Tiles, Strategy Games, Student Instructional Materials, and Teacher Resource Materials. Each item has been described briefly, and current prices (of course, subject to change) have been quoted. Since such matters as postage, handling charges, and discounts depend on the particular order submitted, no attempt has been made to take these into account. It is recommended that schools obtain the latest catalogs of the various suppliers. Suggested grades are indicated merely to help with the buying; since most of the materials are very open-ended, they have uses appropriate to other ages as well as those mentioned.

This bibliography is not intended to be an exhaustive list, but rather a means of helping teachers in their search for appropriate manipulative materials. Although some of the materials do lend themselves to group use with classes, for the most part there is no need for every student to be working on the same kind of apparatus at the same time. It is more desirable for schools to buy a smaller number of a greater variety of materials. It also should be mentioned that not all manipulative materials need to be purchased; with a little ingenuity, marvelous things can be done with buttons, bottle caps, milk cartons, styrofoam, cups, cans, and many other items which we habitually throw away. In fact, many of the commercial items can serve as samples to help teachers and students make their own materials.

A list of suppliers with their addresses is given at the end of the bibliography.

This listing includes many of the fine manipulative materials that have been developed in England. The author wishes to thank Mr. Madison Judson, Headmaster of the Fayerweather Street School, Cambridge, Massachusetts, who was formerly a senior lecturer in England, for his advice on the English materials, and Mr. Burton Harrison, President of Selective Educational Equipment (SEE), Inc., 3 Bridge Street, Newton Massachusetts 02195, for his willingness to distribute English items at very reasonable prices so that they may be readily obtained by schools in this country. Since contacts with some of the English companies were still in process at the time of this writing, those items for which information was incomplete are indicated with and asterisk (*).

BLOCKS

*Lowenfield Poleidoblocs, Set "G" (#INV 630). SEE. (Write for latest catalog.)

Fifty-four wooden blocks of various shapes and sizes, colored red, blue, green, and yellow, and packed in a wooden box with a slide-in lid. In observing, handling, and constructing with these blocks, children gain an intuitive understanding of a large number of mathematical relationships, later to be expressed and understood symbolically. Grades K-4.

**From: The Arithmetic Teacher, October, 1968.

Geo-Blocks Materials (#17489). $28.00; Geo-Blocks Teachers' Guide (#18509). $1.92. Webster Division, McGraw-Hill Book Co.

Various geometric shapes cut out of wood. Suggested activities provide students with many opportunities to gain an awareness of the properties of each solid figure and the interrelationships between various shapes. Grades K-9.

Attribute Blocks Materials (#79896). $9.60; Attribute Blocks Manual (#18410). $1.20; Attribute Blocks Problems (#18462). $4.20. Webster Div., McGraw-Hill Book Co.

The materials, consisting of wooden blocks and cubes, plastic squares with pictures of people on them, colored loops and label cards, a set of problem cards suggesting activities, a set of "creature" cards, and a Teachers' Guide, provide an opportunity for children to deal with problems involving classification and the relationships between classes. Various activities give children experience in distinguishing attributes, in carrying out logical experience in distinguishing attributes, and in gaining better understanding of the properties of sets. Grades K-9.

Dienes Logical Blocks (#Z80007). $19.50; Learning Logic, Logical Games, Z.P. Dienes (#Z80002). $2.45. Herder and Herder.

A set of blocks with games designed to give children experience in distinguishing attributes and carrying out logical operations such as conjunction, disjunction, union, and intersection. Grades K-9.

The Dienes Multi-base Arithmetical Blocks -- M.A.B. (#Z80008). $99.50; Set of Work Cards (#Z80009). $17.50; Additional Sets of Blocks - Base 2 (#Z80016). $9.50; Base 3 (Z80017). $11.50; Base 4 (#Z80018). $17.50; Base 5 (#z80019). $25.00;Base 6 (#Z80020). $29.50; Base 10 (Z80021). $39.50. Herder and Herder.

These materials provide experience in other number systems besides the decimal. The decimal number system is represented as follows: Small cubes represent units; rods of 10 units ("longs") represent tens; slabs of 10 longs ("flats") represent hundreds; and large cubes of 10 flats ("blocks") represent thousands. In the main set, four other number systems (of base numbers 3, 4, 5, and 6 respectively) are represented similarly. For example, in the base 3 system, the longs are composed of 3 units, the flats of 3 longs, and the blocks of 3 flats. Four sets of cards for use with the materials (addition and subtraction, division, multiplication, and problem cards) and a manual are provided with the main set. Each set includes enough material to equip a whole class of normal size. Grades 1-6.

*Shape Dominoes (ES 7713/002). SEE.

Twenty-eight thick wooden dominoes useful for shape- and color-matching exercises. Grades K-3.

*Shape Matching Cubes (#GN 1072). SEE.

A set of six 2-inch cubes designed to help students develop an understanding of size order in a series. A cube has a different geometrical pattern embossed on each face. The pattern is repeated on each cube, but in a different size so that there are 6 patterns in 6 sizes for the child to choose and arrange. Grades K-3.

*Ornabo (#ES 9888/942). SEE.

Set of 36 beechwood cubes, 3cm-5cm, from which children can make abstract designs, pictures of trains, flowers, people, and even the letters of the alphabet. Made in Switzerland. Grades K-3.

ETA Discovery Blocks (#1500). $12.00. ETA School Materials Div.
The fundamental purpose of this set is to present the child with a perceptual program so structured that whatever his capabilities, he may find success as well as a challenge. The set consists of 140 hard maple 3/8-in. thick blocks in 17 shapes, and 15 activity cards graduated from simple to complex, intended to lead the child to self-discovery of basic indormation concerning area, fractions, and spatial relationships. Grades 4-9.

*Prisms and Cubes, SEE.
A collection of precision-cut squares, cubes, and other prisms painted in ten colors and boxed in drawers. This is by far the most powerful set of math blocks being made today, with possibilities ranging from arithmetic computation and multibase work to algebraic topics and solid geometry. Grades K-9.

CALCULATORS AND COMPUTERS

*Colored Plastic Washers. 6 gross in 6 colors. SEE. (Write for latest catalog.)
Plastic washers in six colors (white, yellow, pink, blue, red, and gray) with a hole in the center, to be used for "chip-trading" and other counting activities. Grades K-6.

Abacus Board (#INV 103). $1.25; Abacus Tablets (#INV 104) 20 each of 5 colors, $2.75. SEE.
The operations of addition, subtraction, multiplication, and division must be experienced before they can be understood. This abacus allows children to handle and see the processes for themselves. This abacus, designed by L.G.W. Sealey, can be used for three lines of addition and has a wide variety of basic uses. Grades K-6.

Open-end Abacus Kit (#1-14455). $1.20. Houghton Mifflin Co.
A high-quality base 10 abacus ingeniously designed to form a lightweight compact unit. The abacus base is itself a part of the semi-rigid plastic container in which all of the kit materials (5 rods of uniform size and 45 beads in three colors) are stored when not in use. Each "open-ended" rod will fit securely into one of three receptacles in the abacus base and will accept at most 9 beads. An identical second rod can be fitted on to the base rod to allow for temporary situations arising in renaming. Grades 1-6.

Abacus Supplement. 66¢. Houghton Mifflin Co.
A supplementary package of 15 rods for bases 2, 5, and 8 to be used in the Open-end Abacus. Grades 1-6.

Multi-Base Converter with 100 pegs (20 each of 5 colors), $2.25. SEE.
A multibase converter to illustrate how groups of numbers can be arranged into various bases. Instruction booklet included. Grades 4-9.

*Addo Calculator (#GN 1323). SEE
Designed for primary use showing addition, subtraction, and division up to 249. In tough plastic with instructions. Grades K-4.

*Easy Adder (#INV 431). SEE.
An ingenious peice of equipment, made of colorful plastics, which aids in the teaching of addition and subtraction. Grades 1-6.

SEE "Calculator" (SEE #001), Bulk packed at 10 for $13.00 or $1.40 each. SEE.
This device has been designed to aid in making addition and subtraction understandable processes. Primary school children, interested in the movement of gears and wheels, learn basic relationships of numbers. Some children can follow the workings to gain an understanding of simple calculators. The entire case is transparent. From the front, the student can see the numerals moving into place, while the back clearly shows how the mechanism works -- particularly interesting when the next wheel is advanced. A stylus is provided to facilitate dialing. Grades K-6.

Ten-inch Math Rule (SEE #002), Bulk packed at 10 for $10.00 or $1.15 each. SEE.
A simple slide rule designed for use in elementary schools. The student can add, subtract, multiply, and divide on this one rule which has both a linear and a logarithmic scale. The slide is reversible and the blank back provides an excellent writing surface so that the student can create his own scales. Made of sturdy styrene plastic. Grades 2-9.

Dr. Nim (#G 154). $3.50. Childcraft Equipment Co.
A fascinating, challenging application of computer principles. Children can play Dr. Nim -- a working binary computer -- or a live opponent in a variety of games. Grades 2-9.

Think A Dot (#G 155). $2.50. Childcraft Equipment Co.
This color-pattern game employs a plastic digital computer that changes spot colors when marbles are dropped into three holes on top. Can be played by one or two, and there are many variations including the programming of special patterns. Grades K-7.

Digi-Comp (X 120). $5.95. Childcraft Equipment Co.
A working digital computer that demonstrates the operations usually hidden in electronic circuits. Instructions include simple explanations of computer theory, binary numbers, and programming. Grade 5-9.

Digital Computer Lab (Brainiac) (#1029). $12.50. Lano Co.
A digital computer to be assembled by the students. Grades 6-9.

Analog Computer Kit (#1069). $14.95. Lano Co.
An analog computer to be assembled by the students. Grades 6-9.

CARDS

Mirror Cards (#18418). $10.80; Mirror Cards Teachers' Guide (#18417). $2.32. Webster Div., McGraw-Hill Book Co.
The materials consist of a box of cards, several sets of colorful pictures, four unbreakable plastic mirrors, and a Teachers' Guide. In general, the activity involves trying to match a picture on one card to the picture on another by using a combination of some part of that picture and its reflection in a mirror. Some cards are easy to match; some only seem easy, and many cannot be matched at all. Four children can use one box of materials at a time. Grades K-9.

*Picture Addition (#ES 8830/002). SEE. (Write for latest catalog).
Children are helped to answer the printed sums by arranging small picture counters against a background. The set contains 4 background cards, 12 picture counters, and a supplementary set of sum cards using numbers up to 12. Grades K-2.

*Setsplay (#GN 1318). SEE.
Twelve work cards, 24 games and activities of progressive difficulty, 2 sets of colored shapes - a new method of introducing the concept of sets, the language of sets, and basic elements of logical thought. Two groups of children can work on one set. Grades 4-6.

*Geometric Shape Spotting (#GN 1046). SEE.
A game consisting of 4 boards and 32 cards. For the very young, it can be used purely to help develop the sensory powers of children. Encourages the players to have an awareness of shape and form, and stimulates interest in mathematical properties. Grades K-4.

*Spot the Set (#GN 1158). SEE.
Four boards and 32 cards from which players will discover the qualities of collections through the classification and comparisons of shapes and sets. This game is intended to follow "Geometric Shape Spotting." Grades K-4.

*Geometric Dominoes (#GN 1159). SEE.
A game with 24 domino cards for one or two players, designed to teach quick recognition and identification of geometric shapes, and shape and pattern correspondence. Grades K-4.

*Geometric Matching (#ES 8826/005). SEE.
A card game for the child who has been introduced to geometrical terms. Half of the 56 cards are printed with diagrams illustrating geometrical properties, and the remainder of the pack bears written descriptions of each property. Grades 4-9.

Twin Choice. $1.67 per deck. Holt, Rinehart & Winston.
Pupils sharpen computational skills and win this Hearts-like game by solving mathematical problems given on cards. Eight separate decks give pupils practice on progressively harder problems in addition, subtraction, multiplication, and division. Grades 1-6.

*Come Out Even. Holt, Rinehart & Winston. (Write for latest catalog.)
To play this game of mathematical rummy, students add simple fractions and find common denominators. Shadings and markings on cards help in recognizing equivalent fractions. Two separate decks: Deck A: ½, ¼, 1/8, 1/16; Deck B: ½, 1/3, 1/6, 1/9. Grades 4-7.

Mathematics Using String (#M 11). $1.00. Math Media Div., H & M Associates.
A set of independent exploration cards dealing with distances, thickness, equal lengths, rectangles, index of rotundity, graphing, and open sentences. Grades 3-6.

Independent Exploration Material - Madison Project. Set of 6, $16.26; 4 sets of 6, $15.00 each. Math Media Div., H & M Associates.

Shoe Box Kits, developed under the direction of Dr. Robert B. Davis. These kits contain instruction cards and materials to be used independently by children. Contents: geoboard and workcards, Tower puzzle and workcards, centimeter blocks and workcards, discs and workcards, weights, springs, and workcards, peg game and workcards. Grades 5-8.

CONSTRUCTION

Polyshapes (#INV 202). $13.00. SEE.

For the construction of geometric models, this set consists of 8 plastic templates, 36 sheets of construction cards (20 in. by 25 in.), elastic bands, and 28 work cards. Grades 2-9.

Geo Strips (#INV 141). $8.00; Geo Strip Work Cards (#INV 142). 75¢. SEE.

The Geo Strips are made of strong, pliable polythene in four bright colors. Each strip is drilled and can be joined quickly by brass paper fasteners. By using the special protractor, students can construct accurately angled geometric figures -- and of course many random designs can be built up. The complete set, which is housed in a strong storage box, comprises 68 strips and a box of 144 brass fasteners. There are 11 work cards posing simple and advanced problems. Grades 4-7.

Geo-D-Stix (#M 322). $4.95. Childcraft Eequipment Co.

Color-coded, sturdy, vinyl sticks in various lengths fit into flexible plastic connectors to build all sorts of structures, including the three basic polyhedra. 350 sticks. Instructions provided. Grades 2-9.

Snap Sticks (#M 284). $4.95. Childcraft Equipment Co.

Unbreakable connectors and wheels and hardwood dowels snap together and hold firmly to allow children to construct easily many different structures. 166 brightly colored pieces. Grades K-3.

Moby Linx Construction Kits. $3.95 each; $39.50/doz. Kindry Mfg. Co.

Each set consists of 536 pieces. Specially formed plastic tubes, precision-grooved clips, pliable rubber-band connectors, accessory wires, and simple instructions which can be used to make multidimensional geometric models and decorative mobile objects. Grades 3-9.

Space Spider (#M 360). $3.95. Childcraft Equipment Co.

This "string sculpture" kit makes stunning designs in three dimensions. Special elastic string, in brilliant fluorescent colors, can be threaded through the perforated black panels to make hundreds of fascinating mathematical patterns. Grade 4-9.

Solid Geometry Set (#INV 355). $4.50. SEE.

A gray slotted plastic board provides a base on which to build cubical and pyramidal geometrical figures with the 14 plastic shapes. Grades 4-6.

*Construct-A-Cube Puzzle (#GLN 872). SEE (Write for catalog.)

Set 1 consists of 27 one-inch wooden cubes in natural color. Some of the faces are painted so that they can be put together to form (a) a painted 3-inch cube, (b) an unpainted 3-inch cube, (c) a painted 2-inch cube or (d) an unpainted 2-inch cube. Grades K-3.

Knupperli (#R 040). $1.50. Creative Playthings.
A popular, continental, easy-assembly construction toy providing flexible plastic components for chains, boards, shapes, baskets, and geometric forms. Grades K-2.

DRAWING TOOLS

Kack of 30 Protractors (#PR-61). $13.50. Geyer Instructional Aids Co.
Highly accurate (½"), Two-scale (L to R, R to L) protractors made of pliable non-crack, crystal-clear plastic and housed in a rack which provides a convenient way of checking returns. Grades 4-9.

*Compasses (#INV 253). SEE (Write for latest catalog.)
Senior compasses with a highly polished brass finish. The outstanding unique feature is a screw fastener to secure the pencil. Grades 2-9.

*Oxford Maths Set (#INV 251). SEE.
This comprehensive set housed in a strong metal box consists of the following: solid brass nickel-plated compass; solid brass nickel-plated divider; 6-inch clear plastic ruler; transparent set squares, 45° and 60°; graduated compass pencil; ink-and-pencil eraser; transparent protractor; chemistry stencil; 7-inch pencil; pencil sharpener; blotter; and useful tables. Grades 2-9.

Chalkboard Sets (#E 1970). $11.36. Frederick Post Co.
Complete set with magnetized components containing T-square, straightedge, protractor, right triangle, and compass, all made of sturdy hardwood, with holding knobs and hanger holes. Teacher use, Grades 1-9.

The Geometer. 1-14, $1.10 each; 15-44, $1.00 each; 45-99, 90¢ each; 200 or over, 85¢ each. Academic Industries.
A template, designed by two teachers, that can be used to draw or copy with speed and accuracy the geometric diagrams found in any standard geometry textbook. Fits standard size looseleaf binder. Also has number line; 8-inch ruler; full 360° protractor that can be used in conjunction with the coordinate grid; and a centimeter scale. Grades 5-9.

Invicta Plastics Technical Drawing Models (#INV 150). $3.75. SEE.
Plastic three-dimensional models which allow a wide range of possibilities. More advance drawings are possible by placing two or more models together. Grades 4-9.

*Technical Drawing Book 1 (#INV 180);
*Technical Drawing Book 2 (#INV 181);
*Technical Drawing Book 3 (#INV 182);
*Technical Drawing Book 4 (#INV 183);
*Technical Drawing Book 5 (#INV 184).
SEE. Grades 4-9.

GEOBOARDS

Geoboards can be made for as little as 25¢ each. Requisition 3/8-in. plywood cut into 10 in.-by-10 in. (or 7½ in. -by-7½ in.) squares. As an optional feature that helps young children count the little squares, you may wish to score the 5-by-5 grid, leaving 1-in (3/4 in.) margins and making the grooves 2 in. (1½ in.) apart. Also, so that the rubber bands show up better, you may wish to spray-paint the board black prior to placing the 25 brass, round-headed nails 2 in. (1½ in.) apart with 1-in. (3/4 in.) margins. Making geoboards has been a welcome project of some junior high industrial arts students. Information on commercial geoboards is given below.

The Sigma Geosquare. (See prices below.) Sigma Enterprises.
Full 10 in-by10 in. size makes patterns easy to make and see. Boards are made of ½-in. five-ply board with black plastic perma-ply covering. Grades K-9.

G-1 Geosquare Classroom Kit, $42.00. Includes 30 Geosquares, large assortment of colored rubber bands, complete teacher's manual, pad of Geosquare dot paper.
G-2 Set of 5 Geospuares, $7.50.
G-1M Geosquare Teacher's Manual, $1.00. (Very helpful to buy even if you make your own geoboards.)
G-3A Geosquare dot paper (10 in, by 10 in. with 2-in. scale) 100 sheets, $1.50.
G-3B Geosquare dot paper (10 in. by 10 in. with 1-in. scale) 100 sheets, $1.50.
G-3C Geosquare dot paper (10 in. by 10 in. with ½-in. scale) 100 sheets, $1.50.

Plastics Shapes Board (#INV 148). $1.00. SEE.
The boards are manufactured in unbreakable plastic, hygienic and washable. Pins are at ½-in. centers, and there is a 1-in. grid line incorporated on the board. There are 19-by-19 pins on 9 in-by-9 in. grid size. This board is very versatile. Grades K-9.

Inquiry in Mathematics via the Geo-board, Donald Cohen. Manual and masters, $3.75; Geo-Card Math Lab, boxed, $6.95. Walker Teaching Programs and Teaching Aids.
The manual suggests games and puzzles for the teacher who wishes to explore such topics as counting; even, odd, triagular and square numbers; addition and multiplication of whole numbers and fractions; variables; coordinate geometry; perimeter; area of polygons; congruence; similarity; symmetry, rotations, and reflections. The card lab introduces approximately 150 separate problems and experiments that students can select to work on individually or in small groups. Grades K-9.

MEASURING DEVICES

4-in-1 Ruler (SEE Rule 41). $3.50/doz. SEE.
A handy 12-in. ruler of virtually indestructible plastic -- ideal for classroom use. Features four scales; one side divided into 1/16-in. and 1/10 in. calibrations; reverse side has both a +/- number line and a metric scale (30cm). Markings are recessed, sharp, clean, and accurate for quick and easy student reference. Grades K-9.

*The Yard Rule with Four Faces (#GN 1079). SEE. (Write for latest catalog.)
This rule can be used for measuring in yards, half-yards, feet, and inches. One measurement can easily be converted to the other by turning the rod. Non-toxic enamel finish. Grades K-6.

Meter Stick. $2.65. Geyer Instructional Aids Co.
Metric and English; one side, both edges to millimeters; other side, upper edge only. Inches to eighths; opposite sides reading from same end. Grades 6-9.

*Nonmetallic Woven Measuring Tapes (#INV 648). SEE.
One of the best measuring tapes available in a steel case with folding handle. 33-ft. length. Grades K-9.

Trundle Wheel (#INV 339). $4.50. SEE.
The circumference of the wheel is 1 yard, marked in feet and inches. After each complete revolution, a loud "click" can be heard. Pupils hold the wheel and walk along the distance to be measured, counting the "clicks." Grades 1-8.

Metric Trundle Wheel (#INV 340). $4.75. SEE.
The circumference of this wheel is 1 meter, marked in 5cm and 25cm divisions. An excellent aid in the introduction of the metric system. Grades 3-9.

*Calipers (#ES 9876/138). SEE.
Large beechwood calipers with both English and metric scales. Grades K-4.

Inexpensive 5-in. Vernier Caliper (#40,598). $2.00. Edmund Scientific Co.
Fully enclosed sliding head, inside and outside jaws, depth gauge, sensitive roller adjustment, and locking lever for gauging purposes. Graduated in 1/16 in. to 5½ in. and in 1mm to 140 mm. Vernier reads to 1/128 in. and 0.1 mm. Well constructed of special drawn steel. Complete instructions. Grades 6-9.

Student Micrometer (#60,666). $3.75. Edmund Scientific Co.
An inexpensive beginners' micrometer for students to learn fundamentals of use and technique. Has adjustable anvil. Capacity 0 to 1 in. Accuracy of .001 in. Grades 6-9.

Map Measure and Compass (#E 640). $2.00. Math Media Div., H & M Associates.
Trace along route and obtain distance by multiplying the miles-per-inch factor. Magnetic compass on other side. In Sheath. Grades 4-6.

Pocket Compass (#E 450). $3.60/doz. Math Media Div. H & M Associates.
35mm spun brass case with ring and brass-capped needle. Grades 4-9.

Pedometer (#E 647). $8.00. Math Media Div., H & M Associates.
Easily set stride with crown. Range 0-25 miles with clip for belt. Grades 1-6.

Low-Cost Quality Stopwatch (#40,817). $9.95. Edmund Scientific Co.
Dial reads in 1/10 second. One revolution of long hand every 60 seconds. Small hand registers up to 30 minutes. Has side reset button. 1 jewel. Grades 1-9.

Ten Seconds Timer (#B 108). $12.50. Math Media Div., H & M Associates.
This is a simple device for measuring time. A steel ball rolls down a series of inclined planes and as it drops from one to the other, "beats" seconds. The timer thus measures ten seconds for each run of the ball. Grades 1-3.

F. & C. Wide-Range Thermometer (#60,504). $1.00; (#P-40,775). Pkg. of 6, $5.00. Edmund Scientific Co.
This durable thermometer, filled with red liquid, has stainless steel body etched and filled in black. Marked every 5° from - 20°F. to +220°F. and every 2° from -- 20°C. to +100°C., with numerical values every 10° on both scales. Glass tube 4 in. long, caliborated at 70°F. and 150°F, is mounted with stainless bands and waterproof cement. Size 5¼ in. by 5/8 in. Grades 2-6.

Sunwatch (#S 459). $3.25. Creative Playthings
As its name implies, this is a timepiece without mechanism, needing only the sun to determine the hour. The combination of sundial and compass (with a chart for correcting latitude, longitude, and time of year) is contained in a brass case, pocket size, with full directions. 2 in. by 3 in. Grades 6-9.

Lake and Island Board (#D 982). $9.00. Math Media Div., H & M Associates.
For work in linear measurement, area, and volume. The set consists of a blue board 15 in. square with green "islands," all based on a unit of 1 in., with 30 plastic squares and 50 cubes to the same dimension. The board is used to demonstrate different lengths and distances, rectangles and squares of different sizes, irregular figures, area, equal perimeter with different area, rations 2:1 and 3:1. Instruction booklet included. Grades K-6.

"Invicta" Cliometer (INV 250). $3.75. SEE.
A device to determine the height of buildings by pointing the "gun" at the top of the building and, after looking along the sights, pulling the trigger. A booklet describing its use is included. Grades K-9.

*Angle Meter (#ES 9886/553). SEE. (Write for latest catalog.)
A calibrated angle board which is hinged and can be fixed at right angles to the upright. Grades 4-9.

*Gradient Measure (#ES 9886/553). SEE. (Write for latest catalog.)
The 20-in. base is placed on the ground, and the gradient is measured. Grades 4-9.

*Surveyors Land Chain (#ES 4238/028). SEE.
Good quality chain with brass handles fitted with labels; 2 poles length. Grades 4-9.

Student Transit (#M3). $12.50. Math Media Div., H & M Associates.
A wooden board with sighting tube, protractor, magnetic compass, circular bubble level on top and straight level on side for elevation work. Mounted on a swivel socket attached to a 36-in. dowel. Grades 3-8.

*"Your Foot" Measure (#GN 1319). SEE.
We often use shoe length to measure distance without knowing the size of our feet. This measure is made of wood with a slide and plastic scale. Grades 4-6.

*First Balance (#ES 6225/004). SEE. (Write for latest catalog).
Made of hardwood with 7-in. diameter pans suitable for first experiemnts in weighing. Height 18 in.; overall width 24 in. Grades K-6.

"Invicta" Plastics Mathematical Balance (#INV 100). $5.50; Using the Invicta Balance L.G.W. Sealey (#INV 100). 75¢. SEE.
The balance has 10 stations at each side on both front and back. Each number peg is capable of taking 10 weights, which are placed on the pegs to show relationships between numbers, addition, subtraction, and multiplication. The reverse side of the arm is blank, and white self-adhesive labels are supplied so that each peg may be given fraction, money, length values, etc. Equations may also be solved on this balance. An instruction booklet is supplied. The comprehensive guide provides many illustrations to enable the teacher to make full use of the equipment. Grades K-6.

*Scales with Weights (#ES 9888/470). SEE.
Oval pan scales to weigh up to 7 lbs.; weights ¼ oz. to 4 lbs. Grades K-9.

*Balance with Weights (#ES 9888/497). SEE.
Equal pan scales, 6 in.: cream enameled case; metal base; weights ¼ oz. to 4 lbs. Grades K-9.

Weight Area Volume Set (#A 635). $6.05. Math Media Div., H & M Associates.
This set is valuable in the discovery of relationships of weight, area, and volume. All pieces are related in respect to these properties. The set consists of sixteen ¼-oz. weights, eight ½-oz. weights, four 1-oz. weights, two 2-oz. weights, one 4-oz. weight. All are contained in a strong box with instruction booklet. Grades 1-6.

MISCELLANEOUS ITEMS

Number Line (#1-14478). Set of 10/$2.40. Houghton Mifflin Co.
A 1-in.-by-1-in. transparent, high-quality plastic ruler on which a number line is marked off in twenty-five 1-cm intervals. No numerals are embossed on the line, so that pupils may write on it with a grease pencil or an ordinary No. 1 pencil. Erasures can easily be made by rubbing the plastic with a finger, eraser, tissue, or cloth. Grades 1-9.

Desk Tape Number Line (0 to 10) (#1125). Perforated roll of 40, $1.95; Desk Tape Number Line (0 to 20) (#1126). Perforated roll of 40, $2.95; Desk Tape Positive-Negative Number Line (-10 to +10) (#1127). Perforated roll of 40, $2.95; Desk Tape Fractional Number Line (0 to 1 2/3 showing 1/8, 1/6, 1/4, 1/3, and 1/2). (#1129) Perforated roll of 40, $2.95; Walk-on Number Line (0 to 20) (#1118). $4.50. Instructor Corp. Grades K-6.

Cuisenaire Rods: Classroom Kit (#CX-3D). $57.50; One lot (5 plastic boxes) to extend kit (#CG-&). $11.50; Mathematical Awareness, Part 1, John Trivett, $1.00; Mathematical Awareness, Part 2, John Trivett, $1.00. (Write for catalog for a complete listing.) Cuisenaire Co. of America.

Hundred Chart (#238). $2.95. Instructo Corp.

Develops basic understanding of our number system. Can be used for counting by 1's, 2's, 3's, 5's, 10's, etc. Leads to discovery of number patterns. Chart separates into strips for presentation of sets of 10's. Grades 1-6.

Edmund Graph Stamps. Edmund Scientific Co.

(A) 100 blocks per square inch (#50,255). $3.00. Full pattern measures 3 inches square. Stamp prints one hundred 1/10-in. blocks per square inch.

(B) 16 blocks per square inch (#50,351). $3.00. For plotting in larger scale. Full pattern measures 3 inches square. Stamp prints sixteen ¼-in. blocks per square inch.

(C) Polar Coordinate Graph Stamp (#50,359). $3.00; Large Ink Stamp Pad (#60,134). $1.50; Graph Stamp Set (#70,475). Includes graph stamps (A), (B), and (C) plus stamp pad. $10.00.

Centimeter grid paper (#1-4479). 1-9. 45¢ each; 10 or more, 39¢ each. Houghton Mifflin Company.

Available in pads of 100 sheets -- each 5 in. by 8 in., printed on one side only -- this paper is ruled with grid lines 1 cm apart for an area 10 cm by 10 cm. An area of approximately 3 in. by 5 in. is left blank to provide space for computation or other writing. This paper is useful in the early grades for coloring, cutting, and paper folding. Grades 1-9.

Pickett Slide Rules. Elementary Rule (#N 115). $1.15 each. Pickett.

This is a simple students' rule which contains six scales for basic operations in addition, subtraction, multiplication, division, plus negative and positive numbers and exponential notations. Plastic, with sheath-type case and instruction manual.

7-ft. Demonstration Rule. $25.00
Pickett Projection Rule $10.00

Probability and Statistics Lab Unit, $16.95. Math-Master Labs, Div, of Gamco Industries.

The unit contains various items, such as dice, hexotat, inclined planes, etc., for some excellent experiments suggested in the teacher's book which is included in the set.

Moiré Patterns: Black and white kit. Series A (Deluxe #70,719). $8.50; Color Kit, Series A (Deluxe #60,530). $12.50. Edmund Scientific Co.

Moiré patterns are produced by overlaying precise repetitive figures so that the line elements are nearly superposed. Op-art employs these Moiré patterns in the creation of two- and three-dimensional illustrations. A total of 16 pieces, The Science of Moiré Patterns, and halftone and fiber-glass screens are included. Grades 6-9.

MODELS

Paper Folding for the Mathematics Class, by Donovan A. Johnson, 32 pp. 60¢. NCTM.

Directions for folding paper to illustrate the basic constructions, geometric concepts, circle relationships, products and factors, polygons, knots, polyhedra, and symmetry. Grades 6-9.

Polyhedron Models for the Classroom, Magnus J. Wenninger. 43 pp. 60¢. NCTM.
Directions for constructing various models, with notes on the history and mathematics of polyhedra. Over 40 illustrations and a bibliography. Grades 6-9.

Polyhedron Construction Kit (to accompany book listed above) (#PC-24). $8.95. Math-Master Labs, Div. of Gamco Industries.
Sixty sheets of precut card stock in 6 pastel colors to make 24 multicolored polyhedra. (These cutouts can serve as patterns for students to make very colorful models out of contact paper for mobiles, etc.) Grades 6-9.

Geometric Shapes (#1-4440). Kit of 6, $1.20. Houghton Mifflin Co.
Each kit contains accurate models of a sphere, cube, rectangular solid, cylinder, cone, and pyramid. Being transparent, they enable pupils to visualize the "hidden" lines and surfaces. All of the models are of equal volume (1 cubic inch). The entire set is compactly packaged in a durable, semi-rigid plastic case. Grades K-9.

LaPine Sage Kit (#Z-5000). $8.95; $95.00/doz. LaPine Scientific Co.
Clear-plastic, hollow shapes fit the bases to form the geometrical models. Colored plastic sections fit supporting grooves in the bases to illustrate the geometrical relationships of right sections, slant height, etc. There are 7 basic shapes -- cone, cylinder, hemisphere, cube, square pyramid, triangular pyramid, and triangular prism; 3 multipurpose bases -- square, triangular, circular; and 15 sections to form the intersecting planes. Grades 5-9.

True Equal-Volume Geometric Solids Set (#60,250). $3.00. Edmund Scientific Co.
Set of 6 basic shapes; cube, puramid, sphere, cone, rectangle, cylinder, all made of solid, durable plastic. Each of the solids is a cubic inch, and will displace the same liquid volume. Grades 5-9.

Inexpensive, Large-Size Demonstration Set of Transparent Geometric Solids (#70,314) $39.50. Edmund Scientific Co.
Nineteen-piece set made of transparent plastic on which you can easily mark with a china-marking pencil. Excellent for demonstration purposes, as it allows you to put formulas, dimensions, letters, etc., on each shape. Edges are outlined in red. Height of the largest piece is 4 11/16 in.; the other solids are in exact proportion. Packed in sturdy, hardwood, well-finished box. Grades 5-9.

String Figures and How to Make Them, Caroline F. Jayne. 395 pp. $2.00. Creative Playthings.
This book describes how to make over 100 figures by maneuvering a loop of string. Grades 6-9.

Also refer to the section entitled "Construction."

NUMERICAL GAMES

Kalah (#G104). $7.00. Childcraft Equipment Co.
A stimulating game that uses counters placed in hollows in a hardwood board. Develops interest in arithmetic and teaches counting at various levels as rules are made more difficult. This game can be made conveniently out of egg cartons (providing the 12 pits) and beans (as pebbles). Grades K-9.

Oh-Wah-Ree (#GA-120). $7.95. 3M Co.
A game for two to four players, with a history of 3,500 years and many nations. Players make captures by "sowing" pebbles into the pits. Approximate playing time: 20 minutes. Grades 1-9.

Numero (#INV 189(. $3.75. SEE.
A new number game, the purpose being to "cover up" all the numbers by throwing two dice, adding the addends together and covering up the sum or other addends of the sum. Grades 3-6.

Napier's Rods (#INV 636). $1.50. SEE.
By means of these rods it is possible to do (or check) multiplication by addition. This set contains 10 rods and can be used as a sample for students to make their own. Grades 4-6.

Heads Up. $2.98. E.S. Lowe Co.
A fun game in which numerals and symbols are "thrown" on dice from which the student has to form true mathematical sentences. Grades 5-9.

Quinto (#GA-160). $7.95. 3M Co.
"A Mathematical Scrabble" game intended to be played using multiples of 5, but which can be used to strengthen all of the multiplication and addition facts. Approximate playing time: 40 minutes. Grades 3-9.

The Real Numbers Games. $1.50. Wff'n Proff.
Children roll five dice with numerals and operation signs to obtain as many numbers as possible from the set of naturals, integers, rationals, or reals. Good practice in computation. Grades 4-9.

Yahtzee (#4180-1). $2.29; Yahtzee Refill Pads (#4181-1). 1 pack, $1.00. Miles Kimball co.
The game includes one dice cup, five red plastic dice, twenty bonus chips, one thick score pad and two pencils, and complete simple instructions. The object is to get at least three of every number on the dice. A good chance to discuss some ideas in probability. Grades 5-9.

Arithmecubes (#2156-68). $4.60. Scott, Foresman & Co.
For use at preschool level and in the elementary grades. Sixteen blocks to help children develop the idea of sets, review basic facts, form equations for problem-solving. Blocks show numerals 0 through 9, mathematical signs. Teacher's Instruction Booklet in preparation. Grades K-6.

TUF. $6.75. Tuf.

Based on a number sentence, TUF is a series of games progressing from simple ones involving only the operations of addition and subtraction to more advanced mathematical operations and concepts: multiplication, division, parentheses, fractions, decimals, negative numbers, number systems in other bases, ratio, proportions, percentage, and even exponential powers, fractional roots, logarithms, etc. On signal, two, three, or four players each throw their cubes and work simultaneously and individually to build the longest possible equation. The set contains sixty 3/4-in. colored and impressed cubes, four blank cubes as extra replacements, three timers, one rule book, one container. Grades 5-9.

Krypto. $3.95. Krypto Corp.

Another enjoyable game involving students in numerical computation.

The Arithmetic Drill Games: A Game of Fractions (for Grades 5-9). $5.30; A Game of Multiplication and Division (for Grades 4 and 5). $5.30; A Game of Addition and Subtraction (for Grades 2 and 3). $5.30. Imout.

To stimulate interest in drill work and to simplify the teaching of many of the number facts. Rules are similar to Bingo, and while call cards are preferred for the faster method of playing, there is also a spinning device. Each is a game in which the entire classroom may participate at the same time (48 cards in each game).

*Competitive Fractions Board (#GN 439). SEE. (Write for latest catalog.)

An apparatus consisting of three 16 in.-by-16 in. baseboards, two sets of fractions and twenty-four cards to aid the teaching of fractions. The object of the game is for the three players involved to cover their boards with fractions drawn from the banker board according to instructions on the cards. Grades 1-5.

Equations. $3.50. Wff'n Proof.

Five-game kit that provides practice in elementary arithmetic operations in a variety of number bases. Grades 4-9.

Stocks and Bonds (#GA-170). $7.95. 3M Co.

A stock market game in which each player buys and sells stocks and bonds as he tries to make $1,000. Approximate playing time: 1 hour. Grades 6-9.

PUZZLES

Hindu Pyramid Puzzle. $2.95. World Wide Games.

Known also as the Tower of Hanoi. Object is to shift the "tower" of discs from the center peg to an outside peg, moving only one disc at a time and never putting a larger one on a smaller one. Leads to student discovery and formulation of rules in the form of equations, as the minimum number of moves required is a function of the number of discs in the tower. Seven discs are provided. Grades 4-9.

Ox Bow Solitaire Game. $1.19. Miles Kimball Co.

Known also as the Chinese Shuttle Puzzle. A good puzzle for one person. The object is to put the marbles from the right into the holes on the left and vice versa. Leads to student discovery and formulation of rules in the form of equations, as the number of moves required is a function of the number of pairs of marbles. Grades 4-9.

Two Piece Pyramid Puzzle. $1.25. World Wide Games.
An intriguing puzzle to fit together two precision-cut pieces of cherry wood to make a three-sided pyramid (tetrahedron). Grades 4-9.

Tangram Puzzles (#INV 207). $5.00. SEE.
Each tangram is packed with its own work card in a polythene bag: the two-piece square; the three-peice square; the square and four triangles; the square and eight pieces; the hexagon and ten triangles; and the Chinese Tangram Puzzle. Grades 4-6.

*Tangram Puzzles (#ES 9878/327). SEE. (Write for latest catalog.)
Pack of twelve in assorted colors. Grades 3-9.

Tangram Cards (#17445). $2.70; Tangram Pieces (#17446). $2.70; Tangram Teachers' Guide (#17444). $2.32 Webster Div., McGraw-Hill.
Set includes cardboard borders of various designs into which the pieces can be fit. Grades 2-9.

Tangrams: One Hundred and Thirty Puzzles, by Read. $1.00. Dover Publications
Excellent book to accompany tangram pieces.

What's Your Score Puzzle Assortment (#2240-7). $2.19. Miles Kimball Co.
The assortment is made of wood and contains the famous T puzzle, square puzzle, Hindu Pyramid puzzle, magic squares, and star puzzle. Grades 6-9.

Japanese Tantalizer Puzzle (#2201-1). 59¢. Miles Kimball Co.
Each of four wood blocks has blue, white, red, or yellow dots on its faces. Object is to line them up so that the same color dots appear on each side. Grades 4-9.

Wooden Puzzle Assortment (#2267-1). Miles Kimball Co.
These puzzles are precut finished wood blocks in shaded effect. Each can be taken apart and put together again. There are a 2¼-in. Square block, a 2½-in. ball and a 3¼-in. elephant. Grades 3-9.

Squiggle Wood Contour Puzzle (#2227-1). $2.00. Miles Kimball Co.
Ten pieces in natural wood finish and ten pieces with red finish, all of them contoured to be fit together to form a rectangular solid 6 in. long with checker-board ends. Grades 5-9.

Soma Wood Block Puzzle (#70,262). $2.00. Edmund Scientific Co.
Fourteen cubes make all sorts of fascinating structures which illustrate the value of analyzing the constructions rather than using trial and error methods. Grades 5-9.

Mathematics Games and Puzzle, Kit #1. $6.00; Activity Cards (#SW-1). $2.00; Guide to Teacher's (#GT-1), $3.50. Concept Co.
This kit consists of five small items in plastic: a geoboard, pyramid puzzle, strip puzzle (Chinese shuttle), tangram puzzle, and triagle puzzle. Each puzzle can be bought separately. Grades 2-9.

Tantalizer (#G 132). $5.95. Childcraft Equipment Co.
All you have to do is move a series of objects on the game board while looking in the mirror. A good coordination builder that takes patience and care. Gr. 5-9.

SHAPES AND TILES

Basic Shapes Set (#INV 140). $5.00. SEE.
Young children can learn about the shapes of things by handling and building with the 192 colorful pieces, about 1 in. each in size, in hard-wearing plastic. Grades K-6.

Plastic Mosaic Shapes (#INV 143). $6.25. SEE.
This set of 148 thin plastic mosaic shapes has many uses, including mosaic patterns, picture building, geometric figures, fraction work, and simple algebra. Grades K-6.

*Tilecraft (#GLN 682). SEE. (Write for catalog).
Over 220 plastic tiles for making thousands of patterns and colored designs on a strong pegboard. Grades 1-4.

*Plastic Mosaic (#EKN 870). SEE.
Box of 24 large plastic triangles and squares. Grades K-6.

*Block Mosaic Tiles (#EJKN-43.) SEE.
Seven-inch square containing 40 shaped, polished wooden tiles with colored sheet of designs. Grades K-6.

Linjo (#C 268). $2.00. Childcraft Eequipment Co.
A dramatic new design medium, developed in Europe -- 108 colorful cards that can be combined in an endless number of geometrical patterns. Vigorous colors and harmonious curved lines provide an opportunity for children to be creative. Grades K-3.

Pythagoras (#111); Voo-Doo (#112); Euclid (#113); Kwazy Quilt (#114); Hexed (#115); Tormentor (#116); Sold only in assortments of 3 doz. at $5.69/doz. Tryne Game Div., Kohner Brothers.
Various "take-apart" and "put-together" puzzles, on the idea of Tangram, but involving many kinds of shapes. Also sold in many local stores. Grades 4-9.

Scope (#M 209). $4.95. Childcraft Equipment Co.
Using a basic trapezoid shape, these colorful pieces can be combined in a variety of ways to form hexagons, triangular and diamond patterns with a three-dimensional effect. 144 pieces, a hexagonal tray, instructions, and starting patterns included. Grades K-6.

Three-D Dominoes. (#4799-7). $2.19. Miles Kimball Co.
The game has 45 plastic dominoes in triangular shape. Each has three sections to be matched as regular dominoes. Also there are 6 exciting domino charts included and an interesting solitaire game. Grades 2-9.

Also refer to the previous section entitled "Puzzles."

STRATEGY GAMES

*Compendium of Mathematical Games (#INV 819). SEE. (Write for Catalog.)
Lucky Seven, Pick-up Sticks, Triple Chance, Metal Puzzles, Tantalizing 7, The Challenge, The Maestro, Square, Shake-Numbers, and the Enigma Puzzle. Grades K-9.

Fours (#INV 484). $5.75. SEE.
A game of 3-dimensional tick-tack-toe; the object is to place 4 pegs in a straight line either diagonally, horizontally, or vertically, using any single one or all four surfaces. Grades 4-9.

Qubic (#G 129). $2.95. Childcraft Equipment Co.
A real "plan-ahead" game -- tick-tack-toe in 3 dimensions, for two to six players. Grades 1-9.

On-Sets. $4.00. Wiff'n Proof.
A game on set theory, containing 18 logic cubes, 15 color cards, 2 playing mats, and an instruction manual of 30 different games. Grades K-9.

Configurations (#G-208). $4.00. Childcraft Equipment Co.
A series of intriguing mathematical and geometrical puzzles that will challenge and delight those who enjoy careful reasoning. Based on Professor Dorwart's book, The Geometry of Incidence, a copy of which is included. Gr. 6-9.

Hi-Q Brainbuster (#120). Packaged in lots of 2 doz. at $5.69/doz. Kohner Brothers.
A fun strategy game involving the removal of pegs by jumping schemes in order to try to end with one peg in a particular hole. Also sold in many local stores. Grades 4-9.

Twixt (#GA-110). $7.95. 3M Co.
A stimulating board game with fascinating chess-like strategy of move and countermove. Each player tries to connect his borders with an uninterrupted chain of linked pegs. Either 2 or 4 players. Approximate playing time: 30 minutes. Grades 4-9.

Jumpin (#GA-150). $7.95. 3M Co.
A strategy game for two players or two teams. Board, pawns, and instructions included. Approximate playing time: 20-40 minutes. Grades 4-9.

Chinese Friends. $3.50 World Wide Games.
Unique oriental game for two. If partners use the same color, four may play. Game is also called Reversi. Made of hardwood plywood board, 64 men of two-color rubberized cork. Grades 4-9.

Korean Yoot. $3.50. World Wide Games.
Object of the game is to get 4 men from start to finish first. Moves are determined by throwing 4 Yoot sticks. The game can be played by 2,3,4, or 6 persons. Grades 4-9.

Nine Men Morris (or Mill Game). $4.50. World Wide Games
Game is played by two people; object is to capture the opponent's men. Each time a player gets three of his marbles in a row he can take one of his opponent's marbles off the board. Grades 4-9.

Deluxe Back Up Three. $4.95; Square Back Up Three. $3.95. World Wide Games.
Object is to send 4 marbles all the way around the board by throwing dice. A three sends you back 3 spaces. Two, Three, or four players. Rules can be modified. Grades 3-6.

Reversal (#R 104). $3.95. Creative Playthings.
A strategy game involving a move of "men" from one side of the board to the other. Grades 3-9.

*Hex. SEE. (Write for Catalog)
A plastic board with hexagonal patterns on which is played a strategy game that utilizes principles similar to Twixt. Grades 4-9.

Contack Game (#4456-7). $1.50. Miles Kimball Co.
The game has 36 triangular pieces -- the three sides in contrasting colors, with different numbers. The pieces are divided equally between 2 to 7 players. The first to get rid of all of his triangles is the winner. Grades 2-9.

Pegboard with 8 colors, Boxed (#INV 117). $2.75. SEE.

The Geometry of the Point and Line, Cyril Harris. Area and Volume, Leedham and Packer. The World of Numbers, Norman Beard. A First Book of Fractions, John Clarke. A Second Book of Fractions, John Clarke. Understanding Number Base, John Clarke. Understanding a Contour Map, Kathleen Brooke.
A series of programmed texts designed for pupils with reading age of 10.4 or above. They assume no previous knowledge of their subjects. Tests are included so that the pupil's understanding of the material can be assessed. Grades 5-9.

Exploring Mathematics on Your Own. Glenn and Johnson. $1.12 each. Webster Div., McGraw-Hill book Co.
A series of enrichment booklets to be used by students for individual work or by the class as a whole. Each booklet is complete in itself with exercise sets and answers. Sets, Sentences, and Operations; The Pythagorean Theorem; Invitation to Mathematics; Understanding Numeration Systems; Fun with Mathematics; Number Patterns; Topology -- The Rubber-Sheet Geometry; The World of Statistics; Short Cuts in Computing; The World of Measurement; Adventures in Graphing; Computing Devices. Grades 6-9.

Programed Modern Arithmetic. Raytheon Education Co.
Logic, J. Franklin Fitzgerald and John Blyth. 40 pp. 72¢.
Introduction to Sets, S-1; Set Relations, S-2; and Set Operations, J. Franklin Fitgerald and Raytheon Co.: Jack Starr. 32 pp. each. 72¢ each.
Instructor's Guide, J. Franklin Fitzgerald and John Blyth. 32 pp. 52¢.
Grades 4-6.

TEACHERS' RESOURCE MATERIALS

The Arithmetic Teacher; The Mathematics Teacher. $5.00 a year for one, $8.00 for both publications. Subscription price includes membership in the Council. NCTM.
Very worthwhile journals published monthly eight times a year. October through May by the National Council of Teachers of Mathematics.

Notes on Mathematics in Primary Schools, Members of the Association of Teachers of Mathematics. $4.95. Cambridge University Press.
Full of excellent ideas -- every teacher should own one of these. Grades K-6.

*Mathematics in Primary Schools, Curriculum Bulletin No. 1, The Schools Council. (Write SEE for latest catalog or send $2.00 to Her Majesty's Stationery Office.)
The scope of this English bulletin is wide, as it aims not only at presenting the philosophy of the changes to which reference is made, but also at providing material help for teachers in their own classrooms, and particularly for teachers with large classes in small classrooms. It contains a great deal of helpful material for our elementary teachers. One of the most valuable books available. Grades, K-6.

I Do and I Understand. $1.95; Pictorial Representations. $2.25. John Wiley & Sons.
Two excellent Nuffield Foundation Mathematics Project publications. The former is a guide for teachers and deals with the practical problems of classroom management encountered when children are set free in the classroom to make their own discoveries with concrete materials. The latter is designed to help teachers develop the pupils' ability to use graphs of various kinds. Grades K-6.

Beginnings, $2.50; Mathematics Begins, $2.25; Computation and Structure, $2.50; Shape and Size, $2.50; Desk Calculators, 95¢; How to Build a Pond, $1.75. John Wiley & Sons.
Nuffield Foundation Mathematics Project materials. Grades K-6.

Discovery in Mathematics, and Explorations in Mathematics, by Robert B. Davis. Teachers' Editions. $8.00 each. Addison-Wesley Publishing Co.
Two excellent Madison Project publications full of ideas for enriching and supplementing regular programs. Grades K-9

In-Service Course #1, Madison Project. $3.50. Madison Project. Grades K-9.

First Grade Diary. Robert Hightower and Lore Rasmussen. $2.50. Xerox Education Div.
A lot of ideas for teaching mathematics to primary children. Grades K-2.

Teaching Aids for Modern Mathematics. Paper, $2.95. Holt, Rinehart & Winston.
A source book of ideas for elementary school teachers. Contains items that are inexpensive and easy to make, with directions for their construction and use and suggestions for possible variations. Grades K-6.

Enrichment Mathematics for the Grades. Cloth. $4.00: paper, $2.50. NCTM.
Twenty-seventh yearbook of the National Council of Teachers of Mathematics. Grades K-6.

Number Line, Functions and Fundamental Topics, David Page. $3.80. Macmillan Co.
Ideas for interesting activities for students, written by the Director of the University of Illinois Arithmetic Project. Grades 1-6.

Discovery, Wirtz, Botel, and Nunley. $6.00. Encyclopaedia Britannica Educational Corp.
An excellent multipurpose book for elementary school teachers that contains explanations and illustrations of the unifying ideas constituting the structure of the "new" math. Multilevel Worksheets (24 units, 6 worksheets per unit) and 27 Multilevel Research Activities. Grades K-6.

Amusements in Mathematics. $2.25. Dover Publications.
Foremost British creator of math puzzles is consistently intriguing, witty, and paradoxical in this collection of over 430 mazes, games, and problems involving weighing, packing, age, chess boards, plane figure dissection, etc. Full solutions. Over 450 illustrations.

*Geometry with a Tangram, David Fletcher and Joseph Ibbotson, SEE.
Some valuable applications of the tangram puzzle.

Peas and Particles, Teachers' Guide. $1.20. Webster Div., McGraw-Hill Book Co.
A booklet giving clever ideas of how to get kids involved in such adventures as guessing how many beans it might take to fill a given jar, how many grains of sugar, etc., to fill the same container, and how one might make reasonable approximations.

Contemporary School Mathematics. $1.40 each. Houghton Mifflin Co.
Series 1: Matrices, G. Matthews; Sets and Logic, C.A.R. Bailey; Computers, F.B. Lovis; Shape, Size, and Place, J.A.C. Reynolds.
Series 2: Matrices 2, G. Matthews; Sets and Logic 2, C.A.R. Bailey; Computers 2, F.B. Lovis; An Introduction to Probability and Statistics, A.J. Sherlock. Grades 7-12.

What is Mathematics? $9.00. Oxford University Press.
An excellent resource book for teachers who wish to extend their background in math.

Beginning Mathematics, Z.P. Dienes and E.W. Golding. Herder and Herder.
Part I. Learning Logic, Logical Games (#Z80002). $2.45. An extended introduction to the study of logic using the Logical Blocks (already mentioned in section entitled "Blocks").
Part II. Sets, Numbers and Powers (#Z80004). $1.95. Introduces numbers through the properties of sets and continues up to the idea of "powers."
Part III. Exploration of Space and Practical Measuring (#Z80005). $1.95. Deals briefly with the practical application of number in situations involving measurement, weight, capacity, time, and area, and suggests an early approach to geometry. Because these English publications are so valuable on each topic, they are now available through an American publisher.

Modern Mathematics for Young Children, Z.P. Dienes. (#Z80003), $1.75. Herder and Herder.
This book contains some helpful ideas for teachers at the primary level.

Building Up Mathematics, Z.P. Dienes. (#Z80100), $3.00. Herder and Herder.
An account of the underlying theory.

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